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FERROMAGNETISM AND ITS DEPENDENCE UPON CHEMICAL, THERMAL AND MECHANICAL CONDITIONS.*

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INTRODUCTION.

IT HAS often been pointed out that magnetism has been especially difficult to understand because all of its phenomena have had to be studied statistically, and until a clearer picture of the ultimate magnetic element could be obtained progress in magnetic theory had necessarily to be extremely slow. The great stimulation afforded to electrical theory by the discovery of the electron and the intensive study of its properties illustrates what may be expected in magnetic theory when the properties of the corresponding natural magnetic unit become accessible to investigation. Recent experimental work, to be reviewed in the theoretical part of this paper, makes it extremely probable that this magneton, as it is called, has already been directly observed. It is particularly appropriate at this time, therefore, to survey the existing state of knowledge in regard to magnetism in order to see whether we have not already a respectable body of data which may be used

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in testing the new theory of magnetism toward which we are being forced. Since the foundations of this theory lie in experiments upon single atoms in gases, it will be most worth while to look for confirmation or disproof of the new ideas in experiments of quite a different kind, those upon ferromagnetic solids. This will have another great advantage in the fact that here we have the phenomena of magnetization in so exaggerated a form that the test of any proposed explanation is as severe as possible. In the necessary review of experimental and theoretical work, completeness will be out of the question and the references will accordingly in most cases be to the most recent papers rather than to those first in the field.

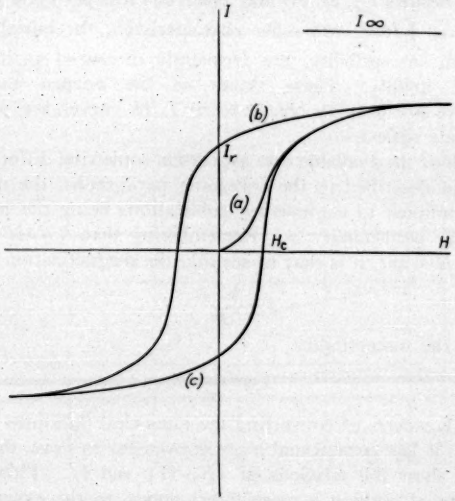
1. FERROMAGNETISM.

As the name implies, ferromagnetism is best defined as the sort of magnetism exhibited by iron. This is characterized by the fact that in iron the magnetic susceptibility, κ, is not constant. The differences between various materials and between the same material in various conditions are often exhibited in the form of curves giving the magnetization per unit volume, I, as a function of the magnetic field H. In general $I = \kappa H$, and if, as in most cases, the vectors I and H are parallel, the susceptibility, κ, is a scalar. If κ were also a constant, as it is not, these curves would all be straight lines through the origin of I and H, differing only in slope. As a matter of fact the curves obtained are of the third or higher degree, possessing in general a point of inflection if H is varied through a sufficiently wide range. There is also, in general, hysteresis, the effect of which is to allow a range in values of I for each value of H, and to allow at least two possible values of slope for I, H curves through each point of the I, H plane which represents a possible state of magnetization and corresponding field strength.[1, 2] The comparison of I, H curves accordingly furnishes a means of comparing at a glance the widest ranges in magnetic behavior. Many attempts have been made to obtain analytical expressions for these curves, but so far without complete success.[3, 4]

Other methods of comparing magnetic properties are only abbreviations of that just described, the ordinates and abscissæ of points on I, H curves representing states arrived at in standard ways, or the areas of certain I, H loops being regarded as charac-

teristic of the sample examined. A too restricted view of these "magnetic characteristics" renders the subject more complicated in appearance than the facts warrant, a comparatively simple distortion of an I, H curve being in general accompanied by changes in all of these "characteristics." We will accordingly, wherever possible, discuss entire I, H curves and give preference in our

FIG. 1.



discussion to experimental work where the course of such curves is clearly defined.

The I, H curves most frequently obtained are shown diagrammatically in Fig. 1. Curve (a), the normal magnetization curve, shows the rise of H and I from a demagnetized state at the point $(0, 0)$, and curve (b) the fall in I as H diminishes from ∞. Since the positive direction of H and I is arbitrary the latter curve and the curve (c) representing the rise in I as H increases from $-\infty$ are symmetrical with respect to the origin, and the two curves enclose that part of the I, H plane in which significant points lie. The closed figure thus formed is the maximum hysteresis loop.

The value of I for $H = \infty$ is the so-called saturation value of magnetization, I_∞; the greatest possible value of I for $H = 0$ is the limiting remanence, I_r; and the highest value of H for $I = 0$ is the limiting coercivity, H_c. Values of remanence and coercivity are usually given, however, for less extensive hysteresis loops, and will then be slightly smaller than the limiting values just mentioned. In such cases the limits of the loop should be stated. Besides I_∞, I_r, H_c and hysteresis loss per cycle per cubic centimetre, $\int HdI$, two other characteristics, the initial and the maximum susceptibility, are frequently measured as indices of magnetic quality. These slopes of the normal magnetization curve are easy to obtain when I, H curves are plotted to appropriate scales.

Much of the available data appears in somewhat different form from that described in the foregoing paragraphs, the quantities most prominent in engineering publications being the induction, B, and the permeability, μ. Remembering that $B = \mu H$ and that also $B = H + 4\pi I$, it is easy to see that the magnetization

$$I = \frac{B - H}{4\pi},$$

and that the susceptibility

$$\kappa = \frac{\mu - 1}{4\pi}.$$

Instead, however, of converting the numerical data into terms of I and H, it has been found more convenient to draw the figures so as to show the relations of $(B - H)$ and H. This has the advantage of making a more direct appeal to the experience of technical men without losing that exactness in the use of the term magnetization which is desirable. In the text I will be used where abbreviation is desired and $(B - H)$ where attention is to be called to numerical values of magnetization in what may be called practical units.

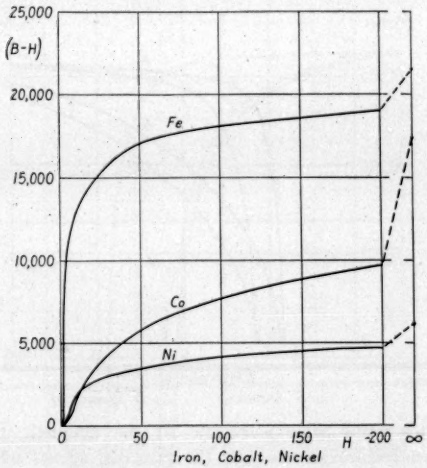
2. CHEMICAL EFFECTS.

The property of ferromagnetism is common to three elements which have so far been obtained in a reasonably pure state. Whether other elements will be found to be ferromagnetic cannot yet be stated with certainty. Few of the rare earths have yet been

tested in the form of pure metals, and it is in this group of elements, if anywhere else than in the iron group, that additional ferromagnetics are likely to be found, for it is here that the highest values of paramagnetic susceptibilities occur in solid compounds.[5, 6, 7, 8]

The three elements so far certainly known to be ferromagnetic are iron, cobalt, and nickel, having atomic numbers 26, 27

FIG. 2.

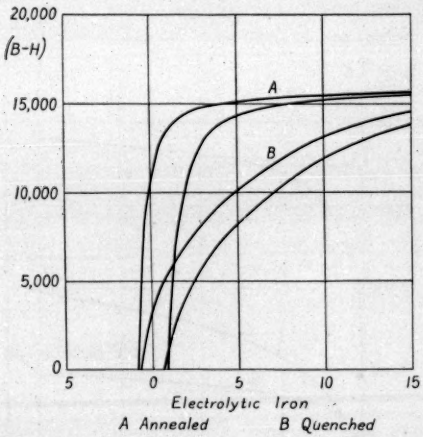


and 28, respectively. It will be noted that the atomic weight of cobalt is higher than that of nickel, but there is no doubt of the correctness of the order assigned.[9] Their relative magnetic properties are indicated in Fig. 2,[10] which shows some normal magnetization curves for annealed wires. The close chemical association of these three elements is one of the most important clues which we have as to the cause of ferromagnetism.

The theory of atomic structure, proposed by Bohr and so brilliantly developed by its author [11, 12] and by others,[13, 14, 15] makes it quite clear why the elements in the range of atomic numbers between 21 and 28 should be the first to show magnetic

properties, but no reason has yet been offered to explain the fact that ferromagnetism in the solid metallic state should be confined to the last three of these eight elements, and should be most conspicuous in the first of these three. This may be taken as the first peculiarity which any adequate theory of ferromagnetism must account for. We will, however, defer attempting an

FIG. 3.



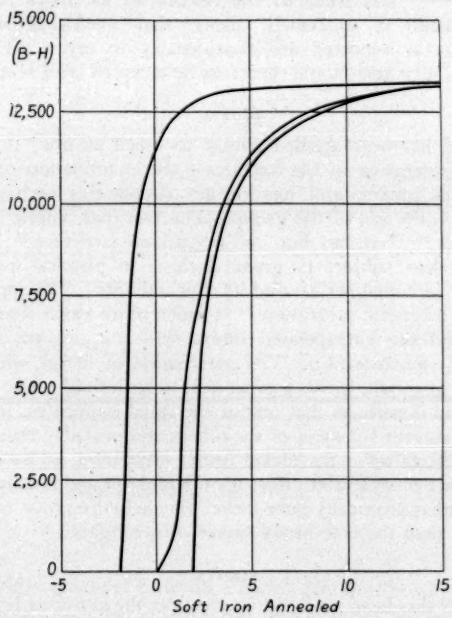
explanation until after a review of the magnetic properties of known ferromagnetics, and the principal agents which vary these properties.

IRON.

In spite of the efforts that have been devoted to the preparation of pure iron, it cannot yet be said that the influence of impurities upon magnetic behavior has been completely eliminated. In the best irons which have yet been tested, the effect of dissolved carbon is still plainly to be seen, as, for example, in Fig. 3,[16] and extrapolation to the properties of what we might call physically pure material, although it has been attempted,[17] is rather unsafe. This is particularly true for low values of H since here, as in many other cases, the value of I_∞ is very

slightly affected by circumstances which profoundly modify the initial portions of the normal magnetization curve and the central portions of the maximum hysteresis loop. The value $(B-H)_{\infty}=21{,}600$ may be taken as typical for the best soft iron,[18, 19, 20] but one observer [21] has obtained 22,600. Both high and low magnetizations are important from the theoretical side,

FIG. 4.



but owing to the variable results of experiments in low fields this region has been relatively neglected and the data collected have not been so fully discussed.[22] It is, on the other hand, in this very sensitiveness to external agents that magnetization in low fields is most significant. Fig. 4 [10] shows more clearly than Figs. 2 and 3 how the initial magnetization of iron proceeds, and

the way in which the normal magnetization curve fits inside a hysteresis loop between wide limits.

COBALT.

Little has been published regarding the magnetic properties of cobalt, and very considerable amounts of impurities have usually been present in the samples used. The shapes of the I, H curves obtained,[10, 23] and some of the results on its alloys mentioned below, make it extremely likely that even the saturation values so far reported are considerably in error. The value $(B-H)_\infty = 17{,}400$ should therefore be accepted with reservations.

NICKEL.

Nickel has been studied almost as much as iron, if we omit from consideration in the latter case the examination of steel in its various guises, and has certain simplicities perhaps to be expected in the last of the group. The fact that, unlike iron [24, 25] and cobalt,[26, 27] it has but one crystalline structure,[26] makes it relatively less subject to great changes in physical properties, including ferromagnetism and its concomitants. The approach of nickel to magnetic saturation [23] is much more rapid than that of cobalt, and the extrapolated limit, $(B-H)_\infty = 6150$, is correspondingly more certain. The purification of nickel, while easier than that of cobalt, leaves a great deal to be desired. The best that can be said is perhaps that one of the usual impurities, iron, is an excellent solvent for most of the others and probably reduces their specific effects upon the nickel itself. But iron, as we shall see, has in itself considerable effect upon both high and low magnetizations, so that physically pure nickel may actually prove rather less magnetic than the best nickel hitherto investigated.

ALLOYS.

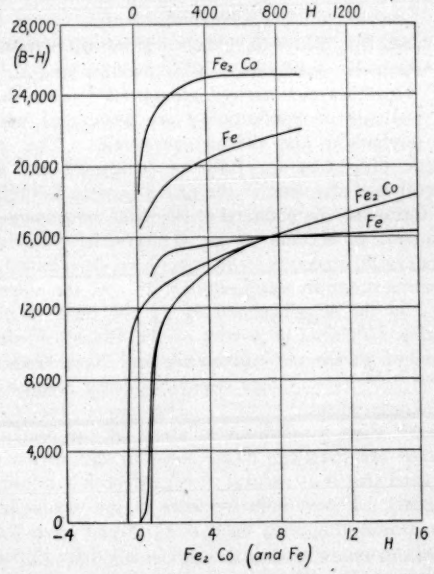
Under this head we will first consider the alloys of iron, cobalt and nickel with each other, and then the alloys of any one or two of them with other elements.

IRON-COBALT.

Among the binary alloys of ferromagnetic elements the iron-cobalt series exhibits one of the most striking anomalies to be found.[28, 29] An alloy containing roughly two-thirds iron and

one-third cobalt has the highest saturation value of magnetization which has yet been reported,[21] $(B-H)_\infty = 25{,}700$. Fig. 5 gives the normal magnetization curves for iron and for this alloy, both being of the highest purity yet attained. The integral ratio empha-

FIG. 5.



sized by the chemical formula, Fe_2Co, may not, however, require to be exact. The high cobalt end of the series suffers from the same objections that have been raised to cobalt itself, and the maximum in I_∞ at so low a cobalt content may be the resultant of opposing influences, the hardening effect of impurities masking a further increase in I_∞ beyond 35 per cent. cobalt. The crystal structures of a series of alloys have been reported [30] and do not suggest the existence of a compound Fe_2Co.

COBALT-NICKEL.

The cobalt-nickel series has been very little studied. Such evidence as we have in regard to its magnetization [31] is probably vitiated by the difficulty of saturating high-cobalt alloys, already emphasized sufficiently in the preceding paragraphs. No anomalous variations have been reported in the values of I_∞.

NICKEL-IRON.

The nickel-iron alloys have two regions of extreme interest from the magnetic standpoint. Near the iron end an extensive series of researches summarized elsewhere [32] have shown how different methods of conditioning are associated with widely different mechanical and thermal properties. The differences in magnetic properties are fully as striking,[33, 34, 2] one form being almost non-magnetic if the nickel content is high enough. The two forms may be obtained at identical temperatures, within certain ranges, on account of the irreversible character of the changes in crystal structure, which has been determined throughout the entire range in composition.[30, 35] At the upper end of this region in the neighborhood of 25 per cent. of nickel, the mechanical and thermal properties of the alloys, of which invar is one, are of paramount importance and have received much attention.[36, 37, 38, 39, 40] Other researches cover a more extended range in composition.[41, 42, 43, 44] The curve for I_∞ may [33] or may not [28] show a minimum at about 25 per cent. of nickel. The so-called non-magnetic nickel steels in this region are ordinarily so hard that it is natural to regard such a minimum as a spurious effect. Recent improvements in the technique of producing enormous magnetic fields [46, 20] should give means for approaching nearer to true saturation in all these difficult cases. We will return to the discussion of these alloys in connection with the effects of temperature changes.

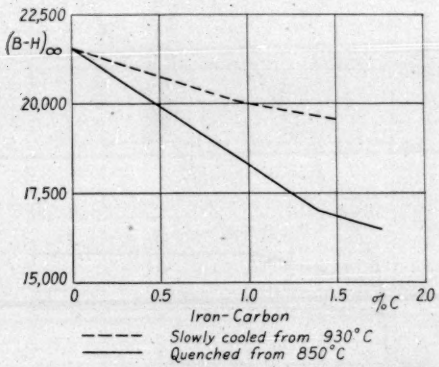
At the high nickel end of the series there exists a very different state of affairs. In this region permalloy [47] exhibits, when properly heat-treated, permeabilities of an order of magnitude higher than any hitherto obtained in either pure metals or alloys, and this in spite of residual impurities by no means negligible in amount. The opportunities here offered for the study of the early stages of magnetization are exceptional. It should be noted, however, that permalloy shows no higher saturation values than are to be

expected from what we know of the pure constituents, so that, except for the convenience of reaching virtual saturation at much lower fields the use of these alloys in the study of the high magnetization region offers no particular advantages.

IRON-COBALT-NICKEL.

No data have been published for pure ternary alloys of iron, cobalt and nickel. The possible range of compositions is so great that it doubtless will be a long time before it can be thoroughly explored unless theory can direct the course of investigation.

FIG. 6.

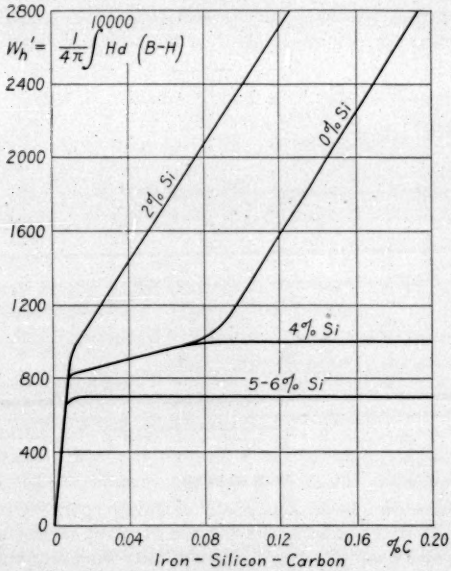


IRON ALLOYS.

The effect of non-ferromagnetic elements upon the magnetic properties of iron has been the subject of many researches, and reviews from the engineering standpoint have been numerous,[48, 18] the technical importance of these properties in many such alloys being enormous. The hardening effect of dissolved carbon has already been referred to, and when carbon exceeds a very small amount the possible occurrence of carbides, principally cementite, Fe_3C, and of free carbon in graphitic form, complicates the problem. The effects of carbon are, therefore, frequently studied.[18, 49, 50] The effect of carbon on $(B-H)_\infty$ is shown in Fig. 6,[18] and is fairly well accounted for by supposing that molecularly dispersed cementite, or, rather, the iron atoms adja-

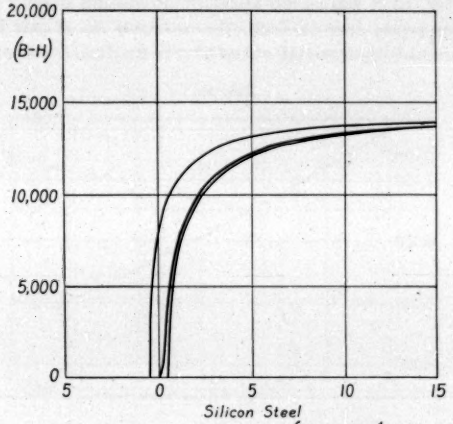
cent to atomically dispersed carbon, are non-magnetizable, but that crystallized cementite has a definite saturation value.[51] Other usual impurities have, in general, smaller specific effects than carbon, and usually have a secondary effect in modifying the distribution of carbon between its three principal forms of occurrence. None except silicon has been added in commercial manu-

FIG. 7.



facture to improve the low field characteristics of the metal. Regarding eddy current loss as a suitable index for magnetic softness, Yensen [17] has very recently published his results upon the effect of silicon in controlling the effect of carbon upon the magnetic properties of iron. Fig. 7 taken from his paper is a smoothed picture of his conclusions. Figs. 8 and 9, taken from Gumlich,[18] show properties of less carefully purified materials. The effect of aluminum is similar to that of silicon but is less

FIG. 8.



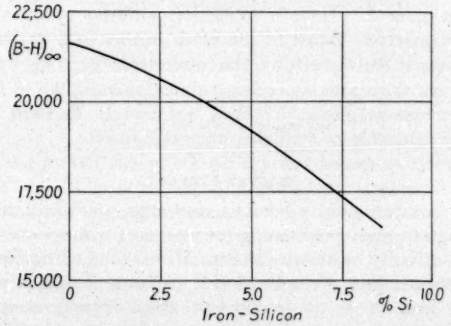
Silicon Steel

Annealed 800° C

Si	4.09%
C	0.07%
P	—
S	0.01%
Mn	0.10%

$H_c = 0.47$, $(\mu - 1)_{max} = 7500$

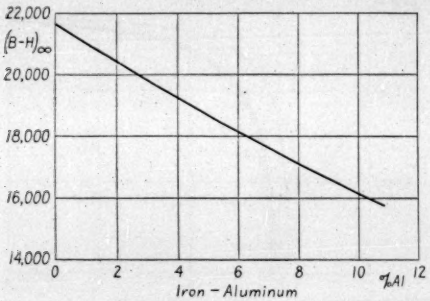
FIG. 9.



Iron - Silicon

easily regulated, aluminum tending to segregate more than silicon does. Fig. 10 [18] shows the effect of aluminum upon $(B-H)_{\infty}$. The advantages derived from the addition of silicon include a great increase in electrical resistivity, a matter of importance in

FIG. 10.



alternating current power transformers and similar devices in which eddy current losses must be low.

MANGANESE STEEL.

One of the most interesting series of iron alloys, from the present viewpoint, is that which it forms with the adjacent element, manganese. Here a moderate addition produces a non-magnetic material. Most of the work in this field has been done with material furnished by the discoverer.[52] The indications here are that the higher manganese alloys, as indicated in Fig. 11,[18] are really non-magnetic,[53, 54] and not merely so hard as to be difficultly saturable in available magnetic fields.

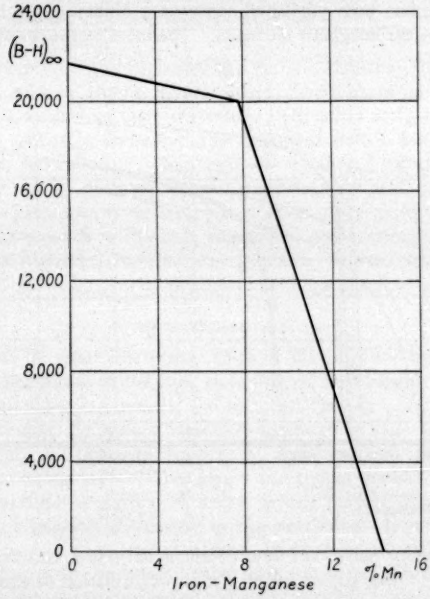
MAGNET STEELS.

The addition of moderate amounts of chromium [55] or tungsten [56] to steel produces great increase in magnetic hardness without seriously reducing the saturation value of magnetization, so that these alloys are suitable for permanent magnets. When the base material is an iron-cobalt alloy of approximately the composition to give the highest saturation, the results are still more striking, the coercivity becoming several times greater than

in any other material yet available. Results with two different steels [57, 58] are compared in Fig. 12.

In all the steels mentioned in the last two paragraphs, considerable amounts of carbon have always been present, so that it

FIG. 11.



is unsafe to assign the effect of any hardening element to that element alone. The preparation of pure binary alloys is particularly difficult in these cases because of the low purity of the available supplies of manganese, chromium, vanadium, tungsten and similar substituents. It will suffice, therefore, to say that the effects of boron,[21] titanium,[59] vanadium [60] and cerium [61] have been studied among others.

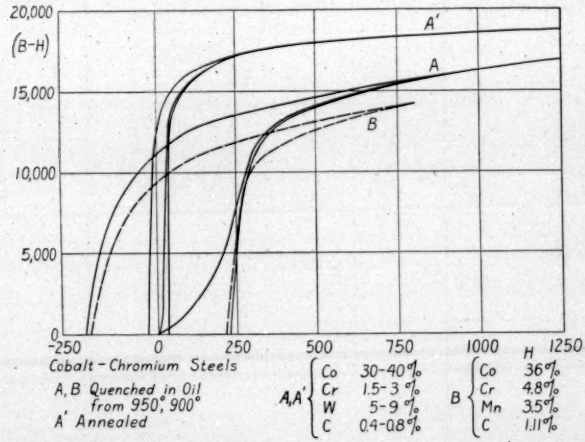
The effect of small amounts of iron upon the magnetic properties of nearly pure non-ferromagnetic metals may also be men-

tioned at this point, since it is usually included in any complete study of a binary system. The name " metamagnetism " has been suggested for the phenomena observed,[62] which are often troublesome in researches on paramagnetism.[63]

COBALT ALLOYS.

Little has been published concerning binary alloys of cobalt with non-ferromagnetic elements. The most recent review [64] lists

FIG. 12.



a good many pairs, but few of them have been studied at all critically. Other papers which contain information of equal value have been omitted or are more recent.[65, 66] As far as the data go the series so far studied is similar in general to the iron-manganese series, only a limited range of any series showing any ferromagnetism at all, and compounds being suspected in many cases.

NICKEL ALLOYS.

Of all the binary alloys of nickel, those which it forms with copper have been most thoroughly studied,[67, 68, 69, 65] as being typical of an unbroken series of solid solutions.[70] The alloys become non-

magnetic at ordinary temperatures at about 30 per cent. of copper, I_∞ remaining, however, nearly proportional to the nickel content. A few observations have been made upon more complex alloys having a nickel-copper base, particularly on the natural alloy known as monel metal or its artificial analogues.[59, 71] Some other nickel alloys have also been made and tested,[59, 64, 65, 66, 67, 72] without disclosing any unexpected properties.

GASES.

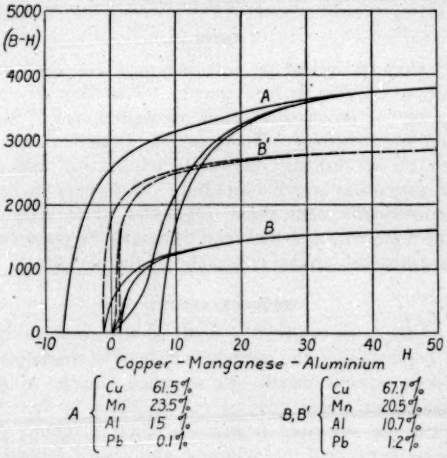
Gases may be sorbed by ferromagnetic elements and alloys, profoundly modifying their properties. The effect of hydrogen is especially patent in electrolytically deposited iron,[19] but it may also affect hard steels.[73] Nitrogen has a marked effect, but is not taken up in ordinary processes of heating and cooling.[74] Except as gases may remove carbon or other elements by forming volatile compounds with these impurities, their effects may be treated as transient, for they can generally be removed to any necessary extent by proper treatment of the samples to be tested.

HEUSLER ALLOYS.

One of the most interesting fields of investigation opened up since the beginning of the century is that of ferromagnetic alloys of non-ferromagnetic metals, the so-called Heusler alloys. This subject has recently been exhaustively reviewed by von Auwers,[64] so that only the principal features of their magnetic properties need here be mentioned. Manganese and copper are the essential constituents, but a third constituent, which may be almost any trivalent or tetravalent element, is also necessary. It is no accident, we may be sure, that manganese and copper are adjacent to the ferromagnetic elements on either side. It is natural to suspect that the extra electrons furnished by the variable constituent are utilized in some way by the manganese to simulate the structure of iron, cobalt or nickel. A similar incipient shift of atomic number and its associated magnetic structure has been suggested by Oxley[75] in another connection. Fig. 13[18] gives some I, H curves representative of such alloys, which have a wide range of magnetic characteristics. These characteristics and related mechanical properties are extremely sensitive to changes in methods of preparation and to heat treatment, and no serious attempt has yet been made to improve the metallurgy of the

Heusler alloys to the point where consistent results can be obtained, so that the interpretation of data already available, and of new material,[76, 77] still remains difficult. The maximum values of $(B-H)_\infty$ are so high, however, that if manganese alone is responsible its atoms must have a magnetic moment about as

FIG. 13.



great as that of iron atoms. It has recently been shown that the symmetry of the Heusler alloys when highly magnetizable is, in part at least, cubic.[78]

CHEMICAL COMPOUNDS.

Related very closely to the last subject is the ferromagnetism of chemical compounds. Besides a few oxides and sulphides of the ferromagnetic elements and, suggestively enough, some similar compounds of manganese,[64, 78] there are few compounds in which the paramagnetic susceptibility is known to increase enough with field strength to suggest ferromagnetism of any importance. High values of κ are found here and there among the salts that have been examined, especially at low temperatures,[5, 6, 7, 8] where, on the usual theory, the paramagnetic properties should be most

marked. The necessity for the complete removal of ferromagnetic impurities,[62, 63] and for the employment of very intense magnetic fields, makes the proof or disproof of ferromagnetism in minute degree particularly difficult.

3. THERMAL EFFECTS.

The effects of temperature upon ferromagnetic properties is frequently complicated by the fact that true conditions of equilibrium are rarely attained by metals at ordinary temperatures, so that the first effect of heating is usually to change the nature of the material, more or less, in the direction of a more stable state. Since such irreversible changes are the more difficult to understand, we will first discuss the effects of temperature on samples so normalized that the changes are sensibly reversible. The method of creating the desired initial conditions varies with the material, but generally involves annealing at the highest temperature which does not produce a change of phase, followed by very slow cooling to the temperature of experiment.

(To be concluded.)

The Contact Electricity of Solid Dielectrics. HAROLD F. RICHARDS. (*Phys. Rev.*, Aug., 1923.)—"The literature of tribo-electricity consists largely of lists in which the various solids are so arranged that any one of them becomes positively electrified when rubbed with a material appearing lower than itself. The singular and disappointing characteristic of these lists is that no two of them are alike." After considering the results of his own experiments the author concludes that they show that "previous discrepancies may be ascribed partly to the impossibility of defining the area over which contact actually occurs in the usual experiment of rubbing a hard solid with a soft buffer; partly to an electric effect of compressing an amorphous material; and partly to the fact that in certain cases the relative velocity of two bodies at the moment of contact determines whether or not an effect due to the inertia of the mobile electrons will be sufficiently large to mask the voltaic electrification."

Discs of different materials were prepared with surfaces flat to within half a wave-length of sodium light. In studying the relation between the charge developed by intimate contact and the area of the contact a glass disc was slid across a steel surface and then twisted around through 90° under the pressure of the hand. The amount of friction was found to have no effect on the charge produced. This charge averaged about 20 electrostatic units per square centimetre of contact. The areas used were from 2.22 to 4.99 square, and the charges per unit area came out so consistent that the investigator

takes this quantity as the "characteristic coefficient of the tribo-electric effect." Further experiments lead to the statement: "Thus the charge is independent of the duration of contact; it is not diminished when the residual air between the surfaces is ionized; and within wide limits it has been found to be independent of the amount of friction. . . . We may conclude on this evidence that the only fundamental difference between tribo-electricity and the Volta effect lies in the greater number of points at which contact must take place to produce the characteristic electrification of an insulator."

Coehn has shown that, when a poorly conducting liquid is put in contact with a solid insulator, the substance of higher dielectric constant becomes positive and that the difference of potential between the two is in proportion to the difference of their dielectric constants. Richards finds the same relation to hold with eight pairs of substances examined, provided that for the steel used the value of 3.1 be assumed for the dielectric constant. For this rather astonishing assumption there is, however, a certain justification adduced. Ebonite fails to align itself with the other materials in regard to the relation of Coehn.

"It is well known that compressible dielectrics can be strongly electrified by pressing them against hard substances." Rubber sheet and pararubber were thus pressed against steel, shellac glass, etc. As the pressure was increased the charge per square centimetre rose to a maximum at which it remained constant. Except when the two kinds of rubber were pressed together, both kinds became negative under pressure. The dielectric constant of the material against which the rubber was pressed made at most but a small difference in the charge per unit area.

"In a recent paper it has been shown that a solid dielectric acquires a perfectly definite charge whenever it collides with a metal provided the relative velocity of the two bodies exceeds a certain minimum value at the moment of contact." The author cites four substances that get a negative charge upon collision with steel while they assume a positive charge upon contact and twisting on the same metal. He holds that "collision gives rise to two different effects, one of which is the contact electrification, while the other is a transfer of electrons from the metal to the dielectric, due to the inertia of the mobile electrons. G. F. S.

Absorption of Sulphur from Producer-gas in Open-hearth Furnaces.—J. H. NEAD, of the American Rolling Mills Company, contributes a paper to the February meeting of the American Institute of Mining and Metallurgical Engineers in which he gives results of experiments that show that sulphur, up to about 0.25 per cent., may be absorbed from the producer-gas in open-hearth furnaces, during the melting-down, being in addition to the sulphur in the original charge. The action imposes a greater burden of desulphurization of the slag than if a lower sulphur fuel were used.

H. L.

OSCILLATIONS AND RESONANCE IN SYSTEMS OF PARALLEL CONNECTED SYNCHRONOUS MACHINES.*

BY

H. V. PUTMAN.

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SYNOPSIS.

THE problem of calculating the resonance frequency and power pulsation of a synchronous motor direct connected to reciprocating apparatus is comparatively easy, provided the machine is operated from an infinite system, *i.e.*, a system having constant voltage and frequency.

In this paper a method is developed for calculating the resonance frequency of two synchronous machines electrically in parallel. It is also shown that in a system of n machines in parallel, there are $(n-1)$ resonance frequencies. A general method of calculating these is given. Systems of three and four machines are treated as special examples of the general case. The theory is also given for calculating the actual power pulsations which occur in systems of two and three synchronous machines connected to reciprocating apparatus. With some approximation the theory is extended to four machines and a general method for n machines is suggested.

Simple methods of calculating the correct sizes of flywheels necessary for the successful parallel operation of engine-driven generators are given. These are calculated directly from a knowledge of the resonance frequencies involved.

In general the thesis is a mathematical treatment of the phenomena which occur in connection with hunting of synchronous machines, electrically in parallel and direct connected to reciprocating apparatus. Specific problems have been treated and illustrative examples presented which make quite clear not only the nature and causes of hunting, but also the methods of overcoming it.

* Communicated by Dr. Ernst J. Berg, Associate Editor of this JOURNAL.

INTRODUCTION.

The increasing use of the internal combustion engine in driving electrical machinery and of the synchronous motor in connection with compressors, has turned the attention of many engineers to the problems of oscillation and resonance in systems of parallel connected synchronous machines.

Perhaps the most pertinent reference, to work already done, that can be cited, is a paper by R. E. Doherty and R. F. Franklin.[1] This paper, however, deals only with the problem of a synchronous motor direct connected to a compressor and operating from an infinite system. The differential equation on which the solution depends has been given by Dr. E. J. Berg[2] and other writers of standard textbooks which deal with the problem of hunting. However, credit should be given to Dr. A. R. Stevenson, Jr.,[3] for having first solved the equation completely with a Fourier's series as the impressed force, obtaining, thereby, another Fourier's series which represents the actual displacement of the motor rotor.

Another paper on the same subject has been published by Theo. Schou.[4] This paper deals with many of the practical problems which occur in connection with the application of synchronous motors to compressors and presents short methods and sets of curves from which the correct flywheel can be quickly determined without elaborate calculations.

An attempt to calculate the resonance frequency of two synchronous machines in parallel has been made by R. E. Doherty,[5] but due to an error in his point of view his result is not quite correct. It does, however, give an answer, which is approximately right in case of two machines having nearly the same number of poles. So far as the writer knows, no attempt has been made heretofore to obtain a general solution from which the resonance frequencies in a system of any number of machines could be calculated.

[1] *Jour. Amer. Soc. Mech. Eng.*, Annual Meeting, Dec. 7–10, 1920.

[2] "First Course in Electrical Engineering," Berg and Upson, McGraw Hill, 1916.

[3] See footnote 1, paper by Doherty and Franklin.

[4] "Present Status of Synchronous Motor for Direct Connection to Compressors," *Refrigeration Engineering*, Aug., 1922.

[5] "Natural Oscillating Frequency of Two Synchronous Machines in Parallel," *G. E. Review*, Feb., 1920.

Another paper especially worthy of mention is one by the late J. Fischer-Hinnen.[6] In his paper Mr. Fischer-Hinnen attempted to calculate the power pulsation between two engine-driven generators in parallel. He neglected the effect of the damping torque due to the amortisseur windings because he did not believe that this effect could be taken care of adequately with linear differential equations, and thus obtained in his answer for the permanent condition of oscillation, certain terms which should have disappeared. These terms were harmonics having a frequency equal to the resonance frequency of the two machines. It seems strange that Mr. Fischer-Hinnen did not realize that, in any permanent condition of oscillation which can occur in practice, either in electrical or mechanical systems, it is quite impossible for oscillations to be present which have frequencies other than those contained in the impressed forces.

A great deal of unpublished work has been done by Dr. A. R. Stevenson, Jr., in connection with these and allied problems. He was the first to set up the differential equations for the power pulsation between two synchronous machines. His method of solution was rather long and cumbersome, but his final expressions for power pulsation check with those which the writer obtains by putting $p = jn_\omega$ in the original differential equations and solving by determinants.

The following theory of a single machine operating on an infinite system and of the parallel operation of two machines is thus not original with the writer. However, the point of view used in solving these equations is new and these examples serve as a logical introduction to the more elaborate theory involved in the parallel operation of three or more machines, developed later in the paper.

CHARACTERISTICS OF SYNCHRONOUS MACHINES.

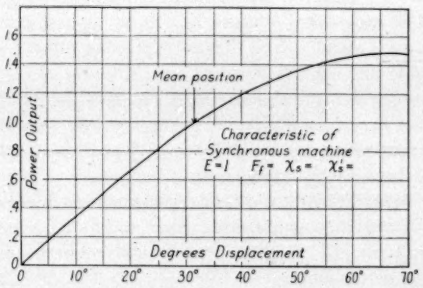
Synchronizing Torque.—It is a well-known fact that when an alternator is loaded its field structure moves forward with reference to its terminal voltage or the voltage of the bus to which it is connected. Similarly, the rotor of a synchronous

[6] "Hunting of Engine Driven Generators in Parallel," *London Elect.*, Aug. 18, 1922.

motor moves backward with reference to its terminal voltage when the motor is loaded mechanically. In either case the angle by which the rotor is displaced from the terminal voltage is called the angular displacement of the machine and is denoted in this paper by the symbol ψ. If the output of an alternator, connected to a constant potential bus and operating under constant excitation, is plotted as function of the displacement of the rotor, a curve similar to that shown in Fig. 1 is obtained.

It can be seen from the curve that if the rotor oscillates about its mean position, the instantaneous power changes with the change

FIG. 1.



of position, but the output per degree displacement remains practically constant as long as oscillations are small. This factor, output per degree displacement, or as is used in this paper, output per mechanical radian displacement, is a very important constant and is denoted by P_o. Thus $P_o = \frac{dP}{d\psi}$. It should be noted that P_o is different at different loads since the curve cannot be considered a straight line over the whole range. P_o also differs with the excitation, terminal voltage, degree of saturation and other factors. The calculation of P_o under different conditions is not within the scope of this paper. It can be done by anyone who thoroughly understands the theory and design of synchronous machines.

Knowing P_o, the synchronizing torque T_s, which is defined as the change in the torque per mechanical radian change in the

displacement of the rotor, can be calculated since the speed is practically constant. Thus:

$$T_s = 3520\left(\frac{P_o P}{S}\right) \text{ foot pounds}$$

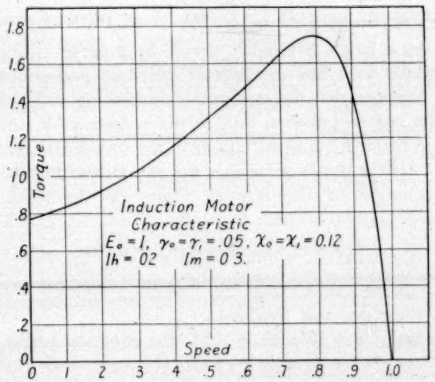
where

P = number of poles.
S = speed in R.P.M.

Neglecting the losses, this is the change in the torque, furnished by the prime mover per mechanical radian change in the angular displacement of the rotor.

Damping Torque.—Practically all synchronous motors and engine-driven generators have amortisseur windings or squirrel

FIG. 2.



cages in the pole faces. These windings produce induction motor characteristics. That is, they produce induction motor torque whenever the rotor tends to speed up or slow down with reference to the rotating electrical field, the torque being produced in a direction tending to prevent further speeding up or slowing down.

A typical induction motor characteristic curve, showing torque as function of speed, is given in Fig. 2. The curve is a straight line in the region around synchronous speed. From this characteristic, the damping torque per mechanical radian per second, slip can be calculated. This constant is called T_d. Its value is usually from ½ to 1½ per cent. of T_s.

RÉSUMÉ OF SYMBOLS.

P_o = Change in k.w. output per electrical radian change in the displacement.
T_s = Synchronizing torque per mechanical radian displacement.
T_d = Damping torque per mechanical radian per second, slip.
P = Number of poles.
S = Speed in R.P.M.
I = Moment of inertia = $WR^2 \div 32.2$, where W is given in pounds and R is the radius of gyration in feet.
θ = Angular position of the rotor (or nominal voltage).
φ = Angular position of the electrical field corresponding to the terminal voltage.
ψ = Angular displacement of rotor ahead of the electrical field, = $(\theta - \varphi)$.
$f(t)$ = Fourier's series representing the engine torque = $a_o + \Sigma\,(a_n \sin n\omega t + b_n \cos n\omega t)$.
ω = Angular velocity of lowest harmonic in $f(t)$.

SYNCHRONOUS MACHINE CONNECTED TO AN INFINITE SYSTEM.

Consider a single alternator, driven by a reciprocating engine and connected to a bus of constant potential and constant frequency. Suppose that the machine is furnishing full load to the bus but its output pulsates, due to the pulsations in the engine torque, and corresponding variations in the watt-meter reading are produced. These power pulsations are easily determined.

The equation of the torques acting on the rotor of the machine is:

$$T_s(\theta - \varphi) + T_d p\,(\theta - \varphi) + I p^2 \theta = f(t) \qquad (1)$$

in which p is the operator $\frac{d}{dt}$ and I is the combined moment of inertia of the rotor and flywheel.

Substituting $\psi = (\theta - \varphi)$ in (1), the equation becomes:

$$T_s\psi + T_d p\psi + I p^2 (\psi + \varphi) = f(t) \qquad (2)$$

But $p\varphi$ is constant since the frequency of the system is constant. Therefore $p^2\varphi = 0$ and (1) becomes:

$$\left(p^2 + \frac{T_d}{I}p + \frac{T_s}{I}\right)\psi = \frac{f(t)}{I} \qquad (3)$$

where $f(t) = a_o + \Sigma\,(a_n \sin n\omega t + b_n \cos n\omega t)$.

Neglecting the transient, the solution for ψ consists of two parts, one due to the constant term a_o and the other due to the series $\Sigma(a_n \sin n\omega t + b_n \cos n\omega t)$. The first part is the average displacement of the rotor due to the load under which the machine is operating. The second part represents the hunting or the variations in the angular displacement due to the harmonics in $f(t)$.

Only the second part is of practical interest as it is this part upon which the power pulsation depends and consequently it is this part which determines whether satisfactory operation can be obtained

PLATE 1.

Power Pulsation in One Synchronous Machine—Connected to an Infinite System.

Gen. rating—44 p.—100 k.w.—125 k.v.a.—.8 p.f.—164 R.P.M.—240 v.—60~

$P_o = 260$ | $\omega = 8.586$	WR^2 of Mot. = 15,000
$T_s = \frac{2625 \text{ H.P.} \times P_o \times P}{\text{R.P.M.} \times \text{k.w.}} = 2.454 \times 10^5$	WR^2 of Flywheel = 612,000
	Total $WR^2 = 627{,}000$
$T_d = \frac{1}{2}$ per cent. $T_s = .0123 \times 10^5$	$I = WR^2 \div 32.2 = 19{,}472$

$a_0 = 4750$ | n	1	2	3	4
$A_n = 100\ (a_n \div a_0)$	−208.42	369.89	−336.84	−156.42
$B_n = 100\ (b_n \div a_0)$	−178.95	73.68	10.11	−41.26
$Q_n = \sqrt{A_n^2 + B_n^2}$	274.90	376.90	336.84	161.85
$In^2\omega^2 \div T_s$	5.85	23.4	52.75	93.80
$M_n = \left(1 - \frac{I_n^2\omega^2}{T_s}\right)$	−4.85	−22.4	−51.75	−92.80
$N_n = T_d n\omega \div T_s$	.043	.086	.129	.172
$D_n = \sqrt{M_n^2 + N_n^2}$	4.85	22.4	51.75	92.80
$\psi_n = Q_n + D_n$	56.70	16.83	6.52	1.75
$B_n \div A_n$	.859 $\mp$	.199 $\pm$	.03 $\mp$	.264 $\mp$
$X_n = \text{arc tan } B_n/A_n$	220.6°	11.3°	178.3°	195.°
$N_n \div M_n$	.0089 $\mp$	.0038 $\mp$	.0025 $\mp$	.0019 $\mp$
$Y_n = \text{arc tan } N_n/M_n$	179.5°	179.78°	179.8°	179.88°
$\alpha_n = X_n - Y_n$	41.1°	−168.5°	−1.5°	14.9°

Amplitude of series $\psi_n \sin(n\omega t + \alpha_n) = Y = 127.6$

Per cent. power pulsation* $= Y \frac{\text{I.H.P.}}{\text{Rated H.P.}} = 140.9$ per cent.

* Percentage based on k.w. rating of machine.

in any particular case. The transient part of the solution which corresponds to the oscillations of the machine, when it is first switched on to the bus, is of little interest, as these oscillations

quickly die out. It can, however, be calculated quite easily in a simple case such as this.

The permanent hunting is easily obtained by putting

$$p = jn\omega$$

n taking on different values corresponding to the different harmonics in $f(t)$. It is more convenient to put just

$$p = j\omega$$

and remember that ω has different values corresponding to the different harmonics. This eliminates the necessity of carrying summation signs through all the equations. In the final solution ω may be replaced by $n\omega$, to retain the definition of ω as the angular velocity of the lowest harmonic. Putting $p = j\omega$, equation (3) gives:

$$\left[\left(\frac{T_s}{I} - \omega^2\right) + j\frac{T_d}{I}\omega\right]\psi = \frac{f(t)}{I} \tag{4}$$

Expressed vectorially $f(t) = a_n + jb_n$ and hence

$$\psi = \frac{\frac{1}{I}(a_n + jb_n)}{\left(\frac{T_s}{I} - \omega^2\right) + j\frac{T_d}{I}\omega} \tag{5}$$

Let

$$\left(\frac{T_s}{I} - \omega^2\right) = M_n,\ \frac{T_d}{I}\omega = N_n,\ D_n = \sqrt{M_n^2 + N_n^2},\ q_n = \sqrt{a_n^2 + b_n^2}.$$

Making these substitutions (5) becomes

$$\psi = \frac{q_n}{ID_n}\sin\left(n\omega t + \tan^{-1}\frac{b_n}{a_n} - \tan^{-1}\frac{N_n}{M_n}\right) \tag{6}$$

or

$$\psi = \psi_n \sin(n\omega t + \delta_n) \tag{7}$$

in which $\psi_n = \frac{q_n}{ID_n}$ and $\delta_n = \left(\tan^{-1}\frac{b_n}{a_n} - \tan^{-1}\frac{N_n}{M_n}\right)$.

The Fourier's series given by (7) represents the variation in the angular displacement of the rotor of the machine in mechanical radians. The torque pulsation is, therefore,

$$T_s\psi + T_d p\psi = \psi_n[T_s \sin(n\omega t + \delta_n) + T_d n\omega \cos(n\omega t + \delta_n)] \tag{8}$$

The power pulsation is proportional to the torque since the speed is practically constant. Thus:

$$\text{Power pulsation in k.w.} = \frac{.746S}{5250}\psi_n[T_s \sin(n\omega t + \delta_n) + T_d n\omega \cos(n\omega t + \delta_n)] \tag{9}$$

The second term is usually very small and adds at right angles to the first so that it may be disregarded without greatly affecting the maximum value of the power pulsation. Hence approximately the

$$\text{Power pulsation} = \frac{.746S}{5250} T_s \psi_n \sin (n\omega t + \delta_n) \qquad (10)$$

But $T_s = \frac{2625}{.746}\left(\frac{P_o P}{S}\right)$.

Substituting this in (10):

$$\text{Power pulsation} = \frac{P_o P}{2} \psi_n \sin (n\omega t + \delta_n) \qquad (11)$$

For practical work it is more convenient to have a formula for power pulsation, derived on a percentage basis as follows: Neglecting T_d in equation (8), the torque pulsation is approximately

$$\psi T_s = \frac{a_n + jb_n}{\left(1 - \frac{I\omega^2}{T_s}\right) + j\frac{T_d \omega}{T_s}} \qquad (12)$$

Expressed in percentage of the average engine torque, the torque pulsation is

$$\frac{\psi T_s}{a_o} 100 = \frac{A_n + jB_n}{\left(1 - \frac{I\omega^2}{T_s}\right) + j\frac{T_d \omega}{T_s}} \qquad (13)$$

where $A_n = \frac{a_n\ 100}{a_o}$, $B_n = \frac{b_n\ 100}{a_o}$

Let

$$Q_n = \sqrt{A_n^2 + B_n^2},\quad M_n = \left(1 - \frac{I\omega^2}{T_s}\right),\quad N_n = \frac{T_d \omega}{T_s},\quad D_n = \sqrt{M_n^2 + N_n^2}$$

Substituting these values in (13):

$$\frac{\psi T_s\ 100}{a_o} = \frac{Q_n}{D_n} \sin (n\omega t + \delta_n) \qquad (14)$$

This expression also represents the power pulsation in percentage of the indicated horsepower of the reciprocating machine. Let Y = the amplitude of this series. Then the power pulsation in percentage based on the rated k.w. output of the generator is

$$\text{Per cent. power pulsation} = Y \frac{\text{I.H.P.} \times .746}{\text{Rated k.w.}} \qquad (15)$$

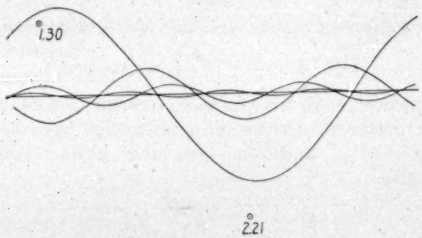
As a specific example illustrating the above theory consider the following problem:

A single cylinder, four-cycle oil engine drives an alternator having the following rating:

44 pole, 125 k.v.a., .8 p.f., 164 R.P.M.
240 volt, 60 ~. $P_o = 260$.

This machine is connected to a bus of constant potential and constant frequency. The WR^2 of the rotor is 15,000 lbs. ft.^{2} and

FIG. 3.



that of the engine flywheel is 612,000. The coefficients of the terms of the Fourier's series representing the engine torque are:

Sines.	Cosines.
$A_1 = -208.42$	$B_1 = -178.95$
$A_2 = 369.89$	$B_2 = 73.68$
$A_3 = -336.84$	$B_3 = 10.11$
$A_4 = -156.42$	$B_4 = -41.26$

$a_o = 4.75 \times 10^5$ foot pounds.

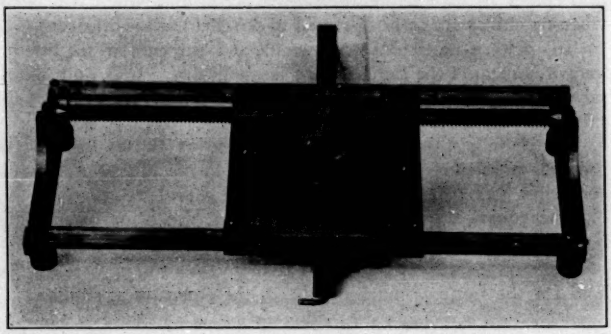
To get the power pulsation, it is only necessary to substitute the above data in formula (14) and obtain the amplitude of the Fourier's series. Substituting this value in equation (15) gives the required power pulsation in percentage based on the generator k.w. rating. The actual calculations are shown in Plate I and the Fourier's series of equation (14), in Fig. 3.

The amplitude of the series is found to be 127.6 and the power pulsation, 141 per cent. This value of power pulsation is obviously much too high and a larger flywheel would be required if the machine were to operate successfully when paralleled with a large system.

For practical work it is not necessary to plot the complete Fourier's series, only the amplitude being required to calculate the percentage power pulsation. The labor involved in obtaining the amplitude of any Fourier's series of four harmonics, is greatly

reduced by the use of a simple instrument, designed and built by the writer for the General Electric Company in 1922. A picture of this instrument is shown in Fig. 4. It will sketch four harmonics of any desired amplitude and phase relation. The harmonics in the Fourier's series for ψ, shown in Fig. 3, were drawn by the instrument. When the harmonics are once drawn

FIG. 4.



Harmonic curve tracer.

it is a simple matter to add them up with the dividers at the maximum and minimum points to determine the amplitude Y.

POWER PULSATIONS BETWEEN TWO SYNCHRONOUS MACHINES IN PARALLEL.

When two engine-driven alternators are connected in parallel and supply power to a steady load, the pulsations in the engine torques cause a transfer of power back and forth between the machines themselves without materially affecting the load. This pulsating power does several undesirable things. It increases the mechanical stresses and heating of the machines and it produces pulsations in the magnitude and frequency of the terminal voltage. Any one of these features may determine the amount of power pulsation which is allowable in any particular case. Experience has shown, however, that a power pulsation of about 60 per cent. is all that may be safely allowed, due to the stresses involved, unless machines are specially constructed. This value of power pulsation will not produce overheating and seldom causes undesir-

able pulsations in either the magnitude or frequency of the terminal voltage. Higher values of power pulsation produce loosening of the coils and wedges in the slots. This results in chafing of the insulation on the conductors and eventually a breakdown of the machine. The life of a machine operating under a high power pulsation, such as 150 per cent., for example, may be limited to a few months of continuous service. The importance of calculating the power pulsations which will occur when machines are to be operated in parallel is thus apparent.

In case two machines are connected through a tie line of known constants the same theory is applicable. It is only necessary to include the line constants in the machine constants in the calculation of P_o for the machines.

The symbols used here are the same as those used in the previous problem. Subscripts (1) refer to machine No. 1, while subscripts (2) refer to machine No. 2.

The equation of the torques acting on the rotor of machine No. 1 is:

$$T_{s_1}(\theta_1 - \varphi_1) + Td_1\, p\,(\theta_1 - \varphi_1) + I_1 p^2\theta_1 = f_1(t) \qquad (1)$$

And for the rotor of machine No. 2, the equation is similar:

$$T_{s_2}(\theta_2 - \varphi_2) + Td_2\, p\,(\theta_2 - \varphi_2) + I_2 p^2\theta_2 = f_2(t) \qquad (2)$$

Substituting $\psi = (\theta - \varphi)$ in (1) and (2):

$$T_{s_1}\psi_1 + Td_1\, p\psi_1 + I_1 p^2\,(\psi_1 + \varphi_1) = f_1(t) \qquad (1a)$$

$$T_{s_2}\psi_2 + Td_2 p\psi_2 + I_2 p^2(\psi_2 + \varphi_2) = f_2(t) \qquad (2a)$$

In these equations there are three unknowns involved, ψ_1, ψ_2 and φ_1. φ_2 is equal to $\frac{P_1}{P_2}\varphi_1$, since the terminal voltage of both machines is the same. The third equation follows from the fact that the sum of the outputs of both machines is equal to the load, which is constant. Thus:

$$S_1(T_{s_1}\psi_1 + Td_1 p\psi_1) + S_2(T_{s_2}\psi_2 + Td_2 p\psi_2) = \text{Constant} \qquad (3)$$

But $S_1P_1 = S_2P_2$ so that:

$$(T_{s_1}\psi_1 + Td_1 p\psi_1) + \frac{P_1}{P_2}(T_{s_2}\psi_2 + Td_2 p\psi_2) = \text{Constant} \qquad (3a)$$

If now only the *variations* in ψ_1 and ψ_2, from their average values, are considered, the constant terms in the right-hand members of equations (1a), (2a), and (3a) disappear. It is only these variations which are of interest since they determine the

power pulsation. Neglecting the constant terms and putting $p = j\omega$ as before, these equations become:

$$\left[\frac{T_{s_1}}{I_1} - \omega^2 + j\frac{T_{d_1}}{I_1}\omega\right]\psi_1 - \omega^2\varphi_1 = \frac{1}{I_1}f_1(t) \tag{1b}$$

$$\left[\frac{T_{s_2}}{I_2} - \omega^2 + j\frac{T_{d_2}}{I_2}\omega\right] - \omega^2\frac{P_1}{P_2}\varphi_1 = \frac{1}{I_2}f_2(t) \tag{2b}$$

$$(T_{s_1} + j\omega T_{d_1})\,\psi_1 + \frac{P_1}{P_2}(T_{s_2} + j\omega T_{d_2})\,\psi_2 = 0 \tag{3b}$$

Let

$$\left.\begin{aligned} \dot{A}_n &= \left[\frac{T_{sn}}{I_n} - \omega^2 + j\frac{T_{dn}}{I_n}\omega\right] \\ \dot{B}_n &= \frac{P_1}{P_n}(T_{sn} + j\omega T_{dn}) \\ \dot{C}_n &= \frac{P_1}{P_n}\omega^2,\ \dot{F}_n = \frac{1}{I_n}f_n(t) \end{aligned}\right\} \tag{4}$$

the dots indicating that the symbols represent vectors. Substituting (4) in the above equations:

$$\dot{A}_1\psi_1 + 0\,\psi_2 - \dot{C}_1\varphi_1 = \dot{F}_1 \tag{1c}$$

$$0\,\psi_1 + \dot{A}_2\psi_2 - \dot{C}_2\varphi_1 = \dot{F}_2 \tag{2c}$$

$$\dot{B}_1\psi_1 + \dot{B}_2\psi_2 + 0\,\varphi_1 = 0 \tag{3c}$$

Solving these equations for ψ_1,

$$\psi_1 = \frac{\begin{vmatrix} \dot{F}_1 & 0 & -\dot{C}_1 \\ \dot{F}_2 & \dot{A}_2 & -\dot{C}_2 \\ 0 & \dot{B}_2 & 0 \end{vmatrix}}{\begin{vmatrix} \dot{A}_1 & 0 & -\dot{C}_1 \\ 0 & \dot{A}_2 & -\dot{C}_2 \\ \dot{B}_1 & \dot{B}_2 & 0 \end{vmatrix}} = \frac{\dot{B}_2(\dot{C}_2\dot{F}_1 - \dot{C}_1\dot{F}_2)}{\dot{C}_1\dot{B}_1\dot{A}_2 + \dot{C}_2\dot{B}_2\dot{A}_1} \tag{5}$$

Substituting for $\dot{A}$, $\dot{B}$ and $\dot{C}$ their equivalents given in (4), the denominator of 5 becomes:

$$\omega^2\left(\frac{P_1}{P_2}\right)^2[D - B\omega^2 + j(C\omega - A\omega^3)] \tag{6}$$

in which,

$$\left.\begin{aligned} A &= T_{d_2} + \left(\frac{P_2}{P_1}\right)^2 T_{d_1} \\ B &= T_{s_2} + \frac{T_{d_1}T_{d_2}}{I_1} + \left(\frac{P_2}{P_1}\right)^2\left(T_{s_1} + \frac{T_{d_1}T_{d_2}}{I_2}\right) \\ C &= (T_{s_1}T_{d_2} + T_{d_1}T_{s_2})\left[\frac{1}{I_1} + \frac{1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right] \\ D &= T_{s_1}T_{s_2}\left[\frac{1}{I_1} + \frac{1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right] \end{aligned}\right\} \tag{7}$$

Also

$$B_2 C_2 F_1 = \left(\frac{P_1}{P_2}\right)^2 \omega^2 (T_{s_2} + j\omega T_{d_2}) F_1$$

and

$$B_2 C_1 F_2 = \left(\frac{P_1}{P_2}\right) \omega^2 (T_{s_2} + j\omega T_{d_2}) F_2$$

So that, due to prime mover No. 1,

$$\psi_1 = \frac{(T_{s_2} + j\omega T_{d_2}) F_1}{(D - B\omega^2) + j(C\omega - A\omega^3)} \tag{8}$$

and due to prime mover No. 2,

$$\psi_1 = \frac{-\frac{P_2}{P_1}(T_{s_2} + j\omega T_{d_2}) F_2}{(D - B\omega^2) + j(C\omega - A\omega^3)} \tag{9}$$

For convenience let:

$$(D - B\omega^2) = N_n, \qquad (C\omega - A\omega^3) = M_n$$

$$T_{d_2}\omega \text{ in (8)} = R_{n_1}, \; T_{d_2}\omega \text{ in (9)} = R_{n_2}$$

It may be well to recall here that in equation (8), ω is really $n\omega$ where ω is the angular velocity of the lowest harmonic in $f_1(t)$ while in equation (9), ω is also $n\omega$, ω being the angular velocity of the lowest harmonic in $f_2(t)$.

Also let $$q_{n_1} = \frac{\sqrt{a_{n_1}^2 + b_{n_1}^2}}{I_1} = \text{modulus of } F_1,$$

and $$q_{n_2} = \frac{P_2}{P_1}\frac{\sqrt{a_{n_2}^2 + b_{n_2}^2}}{I_2} = \text{modulus of } F_2.$$

Equations (8) and (9) may now be written:

Due to prime mover No. 1,

$$\psi_1 = \frac{q_{n_1}\sqrt{T_{s_2}^2 + R_{n_1}^2}}{\sqrt{M_n^2 + N_n^2}} \sin\left(n_1\omega_1 t + \tan^{-1}\frac{b_{n_1}}{a_{n_1}} + \tan^{-1}\frac{N_{n_1}}{M_{n_1}} - \tan^{-1}\frac{T_{s_2}}{R_{n_1}}\right) \tag{8a}$$

and due to prime mover No. 2,

$$\psi_1 = \frac{-q_{n_2}\sqrt{T_{s_2}^2 + R_{n_2}^2}}{\sqrt{M_n^2 + N_n^2}} \sin\left(n_2\omega_2 t + \tan^{-1}\frac{b_{n_2}}{a_{n_2}} + \tan^{-1}\frac{N_{n_2}}{M_{n_2}} - \tan^{-1}\frac{T_{s_2}}{R_{n_2}}\right) \tag{9a}$$

These two Fourier's series together represent the variations in the angular displacement of the rotor of machine No. 1, the first series being due to its own prime mover and the second, due to the prime mover of machine No. 2.

The torque variation of machine No. 1, in percentage based on the average torque of the reciprocating machine No. 1, is approximately,

$$\frac{\psi_1 T_{s_1}}{a_{o_1}} 100.$$

This is also the power pulsation in percentage based on the indicated horsepower of reciprocating machine No. 1. From equations (8a) and (9a), this is:

Due to prime mover No. 1,

$$\frac{\psi_1 T_{s_1}}{a_{o_1}} 100 = \frac{Q_{n_1} T_{s_1} \sqrt{T_{s_2}^2 + R_{n_1}^2}}{\sqrt{M_n^2 + N_n^2}} \sin\left(n_1\omega_1 t + \tan^{-1}\frac{b_{n_1}}{a_{n_1}} + \tan^{-1}\frac{N_{n_1}}{M_{n_1}} - \tan^{-1}\frac{T_{s_2}}{R_{n_1}}\right) \tag{10}$$

and due to prime mover No. 2,

$$\frac{\psi_1 T_{s_1}}{a_{o_1}} 100 = \frac{-Q_{n_2} T_{s_1} \sqrt{T_{s_2}^2 + R_{n_2}^2}}{\sqrt{M_n^2 + N_n^2}} \sin\left(n_2\omega_2 t + \tan^{-1}\frac{b_{n_2}}{a_{n_2}} + \tan^{-1}\frac{N_{n_2}}{M_{n_2}} - \tan^{-1}\frac{T_{s_2}}{R_{n_2}}\right) \tag{11}$$

where $Q_{n_1} = \dfrac{100\sqrt{A_{n_1}^2 + B_{n_1}^2}}{I_1}$ and $Q_{n_2} = \dfrac{100\,\dfrac{P_2}{P_1}\sqrt{A_{n_2}^2 + B_{n_2}^2}}{I_2}$

in which $A_n = \dfrac{a_n}{a_{o_1}}$ and $B_n = \dfrac{b_n}{a_{o_1}}$.

The phase at which the two series, (10) and (11), should be added together, depends upon the angle between the cranks of the two prime movers. If the two machines are of different speeds and none of the harmonics in one torque curve are multiples of any of the harmonics in the other, the angle between the cranks is continually changing, and the maximum power pulsation is the sum of the amplitudes of the two series.

If Y_1 and Y_2 are, respectively, the amplitudes of the series (10) and (11), then the power pulsation in percentage based on the k.w. rating of machine No. 1, is:

$$\text{Power pulsation} = \left(Y_1 + Y_2\right)\frac{\text{I.H.P.} \times .746}{\text{Rated k.w.}} \text{ per cent.}$$

where "I. H. P." and "Rated k.w." refer to machine No. 1.

However, if the two machines are of the same speed this value of power pulsation will not occur unless the machines are synchronized with the cranks in the right position. It would be entirely possible to synchronize two identical machines of the same speed with the cranks exactly together so that no power pulsation would result. In this case, however, the two units would act as a single large unit and there might be considerable speed variation.

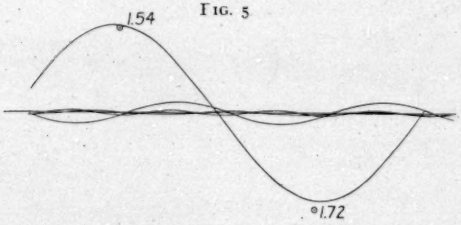
As an example illustrating the above theory, consider the power pulsation between the following engine generator units.

Unit No. 1.

Generator.—40 pole—250 k.v.a.—200 k.w.—.8 p.f.—180 R.P.M.—2300 volt—3 phase—60 cycle. $P_o = 723$.

Engine.—2-cylinder, single acting, 4-cycle, cranks at 360°.

FIG. 5



The WR^2 of the generator rotor is 22,600 and that of the engine flywheel is 752,600. The Fourier's series representing the variations in the engine torque is:

$$f(t) = 393\,[98 \sin\theta - 53 \sin 2\theta + 12 \sin 3\theta + 26 \sin 4\theta + 26 \cos\theta - 12 \cos 2\theta - 13 \cos 3\theta - 12 \cos 4\theta.]$$

The average torque $= a_{o_1} = 9760$.

Unit No. 2.

Generator.—28 pole—120 k.v.a.—96.5 k.w.—.8 p.f.—257 R.P.M.—2300 volt—3 phase—60 cycle. $P_o = 305$.

Engine.—2-cylinder, single acting, 4-cycle, cranks at 360°.

The WR^2 of the generator is 3900 and that of the engine flywheel is 165,000. The Fourier's series representing the variations in the engine torque is:

$$f(t) = 152\,[98 \sin\theta - 53 \sin 2\theta + 12 \sin 3\theta + 26 \sin 4\theta + 26 \cos\theta - 12 \cos 2\theta - 13 \cos 3\theta - 12 \cos 4\theta].$$

The necessary calculations to obtain the two Fourier's series given in equations (10) and (11) are shown in Plate II. A

PLATE II.

Power Pulsation in Two Synchronous Machines—Connected to Reciprocating Machinery—Machines No. 1.

Engine rating—2-cyl., single acting, 4-cycle, cranks at 360°

Generator rating—40 p.—250 k.v.a.—200 k.w.—.8 p.f.—180 R.P.M.—2300 v.—3φ—60~

$P_{o_1}=723$ $\quad \omega_1=18.85$	WR^2 of Gen. $=22{,}600$
$T_1=5250$ H.P. $\div$ R.P.M. $=7817$	WR^2 of Flywheel $=730{,}000$
$T_{s_1}=T_1P_{o_1}\times$poles $\div 2$ k.w. $=56.52\times10^4$	Total $WR^2=752{,}600$
$T_{d_1}=.5652\times10^4$	$I_1=WR^2\div32.2=23{,}373$

$$A=T_{d_2}+\left(\frac{P_2}{P_1}\right)^2T_{d_1}=.394\times10^4$$

$$B=T_{s_2}+\left(\frac{P_2}{P_1}\right)^2T_{s_1}+\frac{T_{d_1}T_{d_2}}{I_1}\left[1+\frac{I_1}{I_2}\left(\frac{P_2}{P_1}\right)^2\right]=39.48\times10^4$$

$a_{o_1}=9760$ $\quad n$	1	2	3	4
$A_{n_1}{}^*=100\ (a_{n_1}\div a_{o_1})$	394.6	-213.4	48.32	104.7
$B_{n_1}{}^*=100\ (b_{n_1}\div a_{o_1})$	104.7	-48.32	-52.35	-48.32
$E_{n_1}=\sqrt{A_{n_1}{}^2+B_{n_1}{}^2}$	407.5	218.8	71.25	115.3
$Q_{n_1}=E_{n_1}\div I_1$	1.743×10^{-2}	$.9361\times10^{-2}$	$.348\times10^{-2}$	$.4933\times10^{-2}$
$M_{n_1}=An^2\omega_1{}^2-Cn\omega_1$	2.298×10^7	20.42×10^7	70.20×10^7	167.4×10^7
$N_{n_1}=Bn^2\omega_1{}^2-D$	13.13×10^7	55.21×10^7	125.35×10^7	223.5×10^7
$R_{n_1}=T_{d_2}n\omega_1$	2.204×10^4	4.408×10^4	6.612×10^4	8.816×10^4
T_{s_2}	11.695×10^4	11.695×10^4	11.695×10^4	11.695×10^4
$H_{n_1}=\sqrt{T_{s_2}{}^2+R_{n_1}{}^2}$	11.90×10^4	12.53×10^4	13.43×10^4	14.66×10^4
$G_{n_1}=\sqrt{M_{n_1}{}^2+N_{n_1}{}^2}$	13.33×10^7	58.95×10^7	143.9×10^7	279.5×10^7
$\psi_{n_1}=Q_{n_1}(H_{n_1}\div G_{n_1})$	1.556×10^{-5}	$.1990\times10^{-5}$	$.0284\times10^{-5}$	$.0259\times10^{-5}$
$B_{n_1}\div A_{n_1}$	$.2653\ \overset{+}{+}$	$.2264\ \overset{-}{-}$	$1.0834\ \overset{-}{+}$	$.4615\ \overset{-}{+}$
$\dagger X_{n_1}=$arc tan B_{n_1}/A_{n_1}	14.9°	192.7°	312.7°	335.2°
$Y_{n_1}=$arc tan N_{n_1}/M_{n_1}	80.1°	69.7°	60.7°	53.2°
$Z_{n_1}=$arc tan T_{s_2}/R_{n_1}	79.3°	69.3°	60.5°	53.0°
$\dagger K_{n_1}=X_{n_1}+Y_{n_1}-Z_{n_1}$	15.7°	193.1°	312.9°	335.4°

Equation for curve of form $y=\psi_{n_1}\sin(n\omega_1t+K_{n_1})$, is

$$y=+\Big\{1.1556\sin(\omega_1t+15.7^\circ)+.199\ \sin(2\omega_1t+193.1^\circ)+.0284\sin(3\omega_1t+313^\circ)+.0259\sin(4\omega_1t+335^\circ)\Big\}\times10^{-5}.$$

Amplitude $=Y_1=3.26\times10^{-5}$

* Reverse signs of coefficients in case of compressor.

† K_{n_1} is usually almost equal to X_{n_1}.

PLATE II—*Continued.*

Machines No. 2.

Engine rating—2-cyl., single acting, 4-cycle, cranks at 360°

Generator rating—28 p.—120 k.v.a.—96.5 k.w.—.8 p.f.—257 R.P.M.—2300 v.—60~

$P_{o_2}=305$ | $\omega_2=26.911$	WR^2 of Gen. = 3900
$T_2=5250$ H.P. ÷ R.P.M. = 2643	WR^2 of Flywheel = 165,000
$T_{s_2}=T_2P_{o_2}\times$poles ÷ 2 k.w. $=11.695\times10^4$	Total $WR^2=168{,}900$
$T_{d_2}=.1169\times10^4$	$I_2=WR^2\div32.2=5245$
$C=\dfrac{T_{d_1}T_{s_2}+T_{s_1}T_{d_2}\left[1+\dfrac{I_1}{I_2}\left(\dfrac{P_2}{P_1}\right)^2\right]}{I_1}$ $=18.00\times10^4$	$D=\dfrac{T_{s_1}T_{s_2}\left[1+\dfrac{I_1}{I_2}\left(\dfrac{P_2}{P_1}\right)^2\right]}{I_1}=900.3\times10^4$

n	1	2	3	4
†$A_{n_2}=100\,(a_{n_2}\div a_{o_1}{}^*)$	152.6	−82.54	18.69	40.49
†$B_{n_2}=100\,(b_{n_2}\div a_{o_1}{}^*)$	40.49	−18.69	−20.25	−18.69
$E_{n_2}=\sqrt{A_{n_2}{}^2+B_{n_2}{}^2}$	157.70	85.70	27.54	44.60
$Q_{n_2}=E_{n_2}\left(\dfrac{P_2}{P_1}\right)\div I_2$	2.105×10^{-2}	1.130×10^{-2}	$.3676\times10^{-2}$	$.5952\times10^{-2}$
$M_{n_2}=An^3\omega_2{}^3-Cn\omega_2$	7.190×10^7	60.43×10^7	205.8×10^7	489.2×10^7
$N_{n_2}=Bn^2\omega_2{}^2-D$	27.69×10^7	113.5×10^7	256.4×10^7	456.6×10^7
$R_{n_2}=T_{d_2}n\omega_2$	3.146×10^4	6.292×10^4	9.438×10^4	12.584×10^4
T_{s_2}	11.695×10^4	11.695×10^4	11.695×10^4	11.695×10^4
$H_{n_2}=\sqrt{R_{n_2}{}^2+T_{s_2}{}^2}$	12.10×10^4	13.28×10^4	14.04×10^4	17.18×10^4
$G_{n_2}=\sqrt{M_{n_2}{}^2+N_{n_2}{}^2}$	28.61×10^7	128.8×10^7	329.0×10^7	669.5×10^7
$\psi_{n_2}=-Q_{n_2}(H_{n_2}\div G_{n_2})$	$.8902\times10^{-5}$	$.1165\times10^{-5}$	$.0157\times10^{-5}$	$.0257\times10^{-5}$
$B_{n_2}\div A_{n_2}$				
$X_{n_2}=$arc tan B_{n_2}/A_{n_2}	14.9°	192.7°	312.7°	335.2°
$N_{n_2}\div M_{n_2}$	3.8512	1.8782	1.2459	.9334
$Y_{n_2}=$arc tan N_{n_2}/M_{n_2}	75.4°	62.0°	51.2°	43.0°
$T_{s_2}\div R_{n_2}$	3.7174	1.8587	1.2391	.9294
$Z_{n_2}=$arc tan T_{s_2}/R_{n_2}	74.9°	61.7°	51.1°	42.9°
$K_{n_2}=X_{n_2}+Y_{n_2}-Z_{n_2}$	15.4°	193.0°	312.8°	335.3°

Equation for curve of form $y=\psi_{n_2}\sin(n\omega_2 t+K_{n_2})$, is

$$y=+\Big\{.8902\sin(\omega_2 t+15.4^\circ)+.1165\sin(2\omega_2 t+193^\circ)+.0157\sin(3\omega_2 t+313^\circ)+.0257\sin(4\omega_2 t+335^\circ)\Big\}\times10^{-5}.$$

Amplitude $=Y_2=1.86\times10^{-5}$ \| $Y_1+Y_2=5.12\times10^{-5}$

‡Per cent. power pulsation $=T_{s_1}(Y_1+Y_2)\dfrac{\text{I.H.P.}^*\times\text{p.f.}^*\times.746}{\text{k.w.}^*}=36.2$ per cent.

* Refers to machine No. 1.
† Reverse signs of coefficients in case of compressor.
‡ Per cent. of normal current of generator or motor No. 1.

sketch of one of the Fourier's series, made by the harmonic curve tracer, is shown in Fig. 5. The calculations show a power pulsation of 36.2 per cent. based on the k.w. rating of machine No. 1. This gives an actual power pulsation of 72.4 k.w.

On the basis of the k.v.a. rating of the smaller machine the percentage is:

$$\frac{72.4}{120} = 60.3 \text{ per cent.}$$

which is just about the allowable limit.[7]

The engine torque curves used in the above calculations were made on the assumption that both cylinders of the two-cylinder engine gave equal amounts of power. If due to some mechanical defect or to an unequal supply of the fuel mixture, one cylinder gives appreciably more power than the other, a harmonic having a frequency equal to half the revolutions is introduced in the torque curve. There are also small intermediate harmonics introduced which in general have very little effect on the power pulsation. The lowest harmonic, however, because of its extremely low frequency is apt to cause considerable power pulsation.

To be absolutely certain of successful operation under adverse conditions, this lowest harmonic should be taken into consideration. The method of making a torque curve for an engine in which one cylinder is giving more or less power than the other is not within the scope of this paper. Once the torque curve is made and analyzed, the power pulsation is calculated in the same manner as above.

(To be continued.)

The Torques and Forces between Short, Cylindrical Coils Carrying Alternating Currents of Radio Frequency. W. A. PARLIN. (*Phys. Rev.*, Aug., 1923.)—"To measure the torque between the two coils, a sensitive dynamometer arrangement was used, which consisted of a fixed coil of eleven turns, ten inches in diameter, and a movable coil of eleven turns, twelve inches in diameter, suspended outside of and free to turn about the fixed coil." The coils were set at a certain angle, the current was made to pass through them. The movable coil, deflected by the force due to the current,

[7] An .8 p.f. machine, 120 k.v.a.—96.5 k.w., would ordinarily be as large if not larger than a 120 k.w. unity p.f. machine, so far as its dimensions, amount of copper in its windings, and mechanical strength are concerned. Sixty per cent. based on the k.v.a. rating is therefore a reasonable limit of power pulsation for an .8 p.f. machine.

was brought back to its original position by twisting its suspending wire attached to a torsion-head.

"For a constant 60-cycle current of 0.60 ampere the torque was measured for a series of angles between the coils, from 10° to 90°." The curve connecting the angle with the torque as measured, and the curve for these same two quantities calculated from the first six terms of Maxwell's equation applying to the case, have at least the same form and otherwise are in fair agreement. "The formula refers to coils of a single turn whereas the coils in the present experiment were of finite length and had several turns of wire.

"In agreement with simple theory the torque between the coils for a constant angle of 30° traversed by a current of frequency 0.4×10^6 cycles (750 metres) varied directly as the square of the current. With the current through the coils in the same sense and the angle between the coils maintained constant, it was found that the torque increased with the frequency of the current. The curve indicates that for the region between 60 cycles and about 700 metres no changes in torque with frequency of the current occurred, but that for the wave-lengths below 700 metres the torque increased in a marked manner with the frequency. This increase in the torque with the frequency was not to be expected from the classical formulas of Maxwell, which contain no frequency term. When the currents through the two coils were in the same sense, the (rate of) change in torque with frequency was found to increase as the angle between the coils decreased. For angles between the coils from 90° to 270°, *i.e.*, the currents being in the opposite sense, the torque did not change with the frequency of the current." When the two coils were placed parallel to each other with the distance between their planes kept constant, the force was again found to vary as the square of the current and to increase with the frequency, when the currents were in the same direction. The radiation of energy may be the cause of the discrepancy between the results of theory and of experiment.

G. F. S.

Treatment of Boiler Feed-water in Highly Overloaded Boilers.—In a pamphlet issued by the H. S. B. W. Chocrane Corporation, D. HENDERSON presents some data and results applicable to the operation of boilers under the modern practice of overloading them far beyond their ratings, procedure made possible by the stoker firing. It makes necessary, however, the best obtainable supply of boiler feed, as the tubes must be protected from scale to prevent tube losses and permit high efficiency when transmitting the much larger number of heat units per square foot of surface. It has repeatedly been demonstrated in practice that while at nominal rating a boiler can operate with even fairly thick scale formation without tube losses, at overloads a comparatively thin scale will cause serious tube losses. With correct softening, turbining of tubes should never be necessary.

H. L.

THE RELUCTIVITY OF THE RECENTLY DISCOVERED MAGNETIC METAL, PERMALLOY.*

BY

A. E. KENNELLY, Sc.D.

Member of the Institute.

In the May, 1923, number of the JOURNAL OF THE FRANKLIN INSTITUTE (vol. 195, No. 5, p. 621), is a very interesting paper by Messrs. H. D. Arnold and G. W. Elmen, on "Permalloy, An Alloy of Remarkable Magnetic Properties." In that paper, which was reprinted in the July, 1923, number of the *Bell System Technical Journal*, comparative curves are given for the magnetic behavior of this wonderful new alloy against Armco iron, in three ways, namely, B *versus* H, μ *versus* H, and μ *versus* B; that is, in the relations of flux density B, magnetizing force H and permeability μ. These curves clearly show the great superiority of permalloy over Armco iron in permeability μ, as a function of either H or B.

It is known,[1] however, that in the cases of soft iron, cast iron and steel, the metallic reluctivity ν, or inverse of the metallic permeability $\mu - 1$, is a rectilinear function of H. That is to say, when ν is plotted as ordinates, against increasing H as abscissas, the diagram forms two straight lines, united below by a smooth bend passing through the point of minimum reluctivity or maximum permeability. The initial value of the reluctivity in soft Norway iron is often commonly about 0.003 oersted-cm. or 300×10^{-5}. It descends steeply along a straight line which would strike the

* Communicated by the Author.

[1] "Magnetic Reluctance," by A. E. Kennelly, *Trans. A.I.E.E.*, Oct., 1891, p. 485, vol. 8.

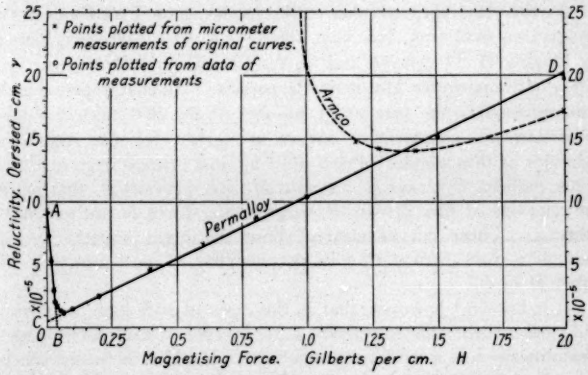
"Variation of Residual Induction and Coercive Force with Magnetizing Force," by R. L. Sanford and W. C. Cheney. Publication of the Bureau of Standards, S. 384, June 23, 1920, 8 pp.

"The Magnetic Reluctivity Relationship as Related to Certain Structures of a Eutectoid-carbon Steel," by C. Nusbaum, W. C. Cheney and H. Scott. Publication of the Bureau of Standards, S. 404, Nov. 26, 1920, 19 pp.

"The Reluctivity of Silicon Steel as a Linear Function of the Magnetizing Force," by J. D. Ball, *Gen. Elec. Rev.*, vol. xvi, No. 10, Oct., 1913, pp. 750–753.

axis of abscissas in the neighborhood of $H = 1$. It bends upward, however (Fig. 2), usually before it reaches $\nu = 10 \times 10^{-5}$, and then follows an upward straight line of the type $\nu = a + bH$, where a and b are constants for the particular quality and sample of iron under test. The constant a, for soft iron, is ordinarily about 10×10^{-5}, and the coefficient b, about 6×10^{-5}. From the bend

FIG. 1.



Reluctivity curves of permalloy and of annealed Armco iron for values of H up to 2.0.

onwards, the metallic flux density B_m, at any given value of H, is obtained from the equation

$$B_m = 4\pi I = \frac{H}{a + bH} \qquad \text{gausses}$$

where I is the "intensity of magnetization." The total flux density B is $B = B_m + H = 4\pi I + H = \frac{H}{a + bH} + H$ gausses.

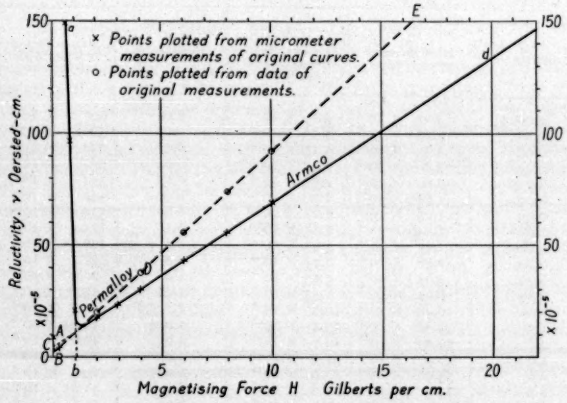
Since the magnetic metals, nickel and cobalt, also manifest this characteristic bilinear reluctivity, when ν is plotted against H, it seemed to be a question of much interest whether the new permalloy would likewise give a bilinear ν-H graph.

The numerical data for the H-B-μ graphs, appearing in the above-mentioned Arnold-Elmen paper, are not given therein; so that the ν-H graphs had to be prepared by making micrometer measurements on the Arnold-Elmen graphs, in their Figs. 2, 3, 4,

5 and 7. The results are, therefore, perhaps somewhat less precise than if the original numerical observations and data had been available. The data in column 3 of Table I, however, have lately been communicated to the present writer by the courtesy of Messrs. Arnold and Elmen, as obtained from their observations of permalloy.

Fig. 1 of this paper shows the results thus obtained for

FIG. 2.



Reluctivity curves of Armco iron and of permalloy up to $H = 22$.

permalloy up to $H = 2$, according to the Arnold-Elmen curves. It will be seen that the initial value of the reluctivity ν is approximately 9.5×10^{-5} oersted-cm. or 95 microöersted-cm., and that it falls along AB, the straight line $(9.5 - 225\ H)10^{-5}$, down to a minimum on the bend of 1.15×10^{-5}, near the extraordinarily low value of $H = 0.06$ gilbert per cm.[2] It then follows CD, the upward straight line $(0.5 + 9.85\ H)10^{-5}$. The bend at the junction of these straight lines is remarkably short and condensed, by com-

[2] If permeability μ is a mere numeric, as some physicists have contended, then H must have the same dimensions as B, and both will be expressible in gausses. Inasmuch, however, as μ is generally regarded as being a physical quantity, and not a simple numeric, it is safer to take H in gilberts per cm., and B in gausses.

parison with the corresponding broken-line bend of the Armco iron, which appears in the upper right-hand quarter of the diagram.

Fig. 2 gives the corresponding reluctivity graph of the

List of Numerical Relations between H and ν as ascertained from Figs. 2, 3, 4, 5 and 7 of the Arnold-Elmen paper of July, 1923.

TABLE I.

1	Permalloy. 2	3
H gilberts/cm.	ν from curves of original paper.	ν from measurement data supplied by Messrs. Arnold and Elmen.
0.0023	9.22×10^{-5}	9.10×10^{-5}
0.01	7.25×10^{-5}	7.25×10^{-5}
0.02	5.33×10^{-5}	5.15×10^{-5}
0.03	3.0×10^{-5}	2.95×10^{-5}
0.035	2.0×10^{-5}	2.05×10^{-5}
0.04	1.845×10^{-5}	1.60×10^{-5}
0.05	1.25×10^{-5}	1.25×10^{-5}
0.06	1.146×10^{-5}	1.15×10^{-5}
0.10	1.43×10^{-5}	1.45×10^{-5}
0.20	2.48×10^{-5}	2.45×10^{-5}
0.40	4.61×10^{-5}	4.55×10^{-5}
0.80	8.64×10^{-5}	8.55×10^{-5}
1.00	10.46×10^{-5}	10.4×10^{-5}
1.5	15.12×10^{-5}	15.1×10^{-5}
2.0	19.84×10^{-5}	19.75×10^{-5}
4.0	37.90×10^{-5}	37.5×10^{-5}
6.0	55.90×10^{-5}	56.0×10^{-5}
8.0	74.0×10^{-5}	74.0×10^{-5}
10.0	91.9×10^{-5}	92×10^{-5}
20	184×10^{-5}	184×10^{-5}
100	926×10^{-5}	920×10^{-5}
$\nu = (9.5 - 225\ H)\ 10^{-5}$	$H = 0$ to $H = 0.04$	
$\nu = (0.5 + 9.85 H)\ 10^{-5}$	$H = 0.06$ to $H = 4$	
$\nu = (0.3 + 9.1 H)\ 10^{-5}$	$H = 4$ to $H = 22$	

TABLE II.

Armco Iron.	
H gilberts/cm.	ν from curves of original paper. oersted-cm.
0.01	397×10^{-5}
0.02	386×10^{-5}
0.03	374×10^{-5}
0.04	362×10^{-5}
0.05	353×10^{-5}
0.06	342×10^{-5}
0.07	333×10^{-5}
0.4	172.4×10^{-5}
0.6	149.5×10^{-5}
0.8	97.1×10^{-5}
1.0	20.9×10^{-5}
1.18	14.8×10^{-5}
1.275	14.2×10^{-5}
1.404	14.04×10^{-5}
2.0	17.0×10^{-5}
3.0	23.1×10^{-5}
4.0	29.5×10^{-5}
6.0	42.85×10^{-5}
8.0	55.8×10^{-5}
10.0	68.9×10^{-5}
20	133.3×10^{-5}
40	252×10^{-5}
60	372×10^{-5}
80	486×10^{-5}
100	596×10^{-5}
$\nu = (350 - 330\ H)\ 10^{-5}$	$H = 0$ to $H = 1$
$\nu = (4.5 + 6.4 H)\ 10^{-5}$	$H = 1$ beyond

Armco iron, as taken from the Arnold-Elmen curves. The descending straight line ab of $(350 - 330\ H)\ 10^{-5}$ would cut the H axis at b, near $H = 1$. The minimum is found near $\nu = 14 \times 10^{-5}$, at $H = 1.4$. It then follows the upward straight line cd of $(4.5 + 6.4\ H)\ 10^{-5}$. The contrast of the permalloy graph ABD,

as far as $H = 4$, is very marked. There is some uncertainty as to the precise direction of the permalloy line beyond D, and the broken line DE, between $H = 4$ and $H = 16.4$, corresponds more nearly to the expression $(0.3 + 9.1\ H)\ 10^{-5}$.

The figures therefore indicate that, at least as far as $H = 4$, which is the important range for permalloy, its bilinear graph of reluctivity is remarkably well maintained and permalloy is no exception to the general rule of magnetic metals in this respect.

Oil Fields in the United States.—The United States is now the largest owner of proved and prospective oil lands. This fact is made known in the thirteenth annual report of the Bureau of Mines covering the service of the bureau for the fiscal year ending June 30, 1923. The government organization is similar to that of the larger private oil companies. Oil is now being produced under lease from public lands and naval oil reserves in California, Colorado, Montana, and Wyoming. During the last fiscal year, wells on public lands, other than Indian lands, yielded nearly thirty million barrels, from which the Government received, as royalty, about five and one-half million barrels, valued at over eight million dollars. In addition nearly one hundred and twenty thousand dollars in royalties on gas and close to sixty thousand dollars on gasoline were obtained. From all sources, the government revenue from control of oil and gas fields reached ten and one-half million dollars. In addition as guardian of the Indians, the Government handles royalties amounting to between twelve and thirteen million dollars yearly.

The report gives a great deal of data concerning fatalities and injuries in mines and the methods of securing greater safety. The problem is a very complicated one. Statistics may seem to indicate a reduction of toll of mine disasters, but careful analysis will often show that such inference is illusory. Thus, in 1907, 3242 men were killed in coal mines, during 1922 the number was 1971. Comparison of these figures would lead to the inference that much advance was being made in securing safety, but the year 1907 was one in which several terrible disasters occurred. In the year 1908, the total dropped to 2445, a figure that was exceeded in seven of the ten years that followed. Had these figures been taken it would be easy to show that coal-mining had become more dangerous. Much of the difficulty in diminishing the fatalities is due to the indifference of the miners themselves. They become neglectful of precautions, often working under dangerous conditions, for instance, under dangerous roofs without troubling themselves to place supports. Deaths by falling roofs constitute the main hazard of those working underground, and the remedy for such perils rests primarily with the workers themselves. H. L.

Rancidity.—The tendency of many fats to acquire disagreeable odor and taste is a matter of common knowledge, but the explanation of the changes producing these properties has not been given. Chevreul, in the early part of the nineteenth century elucidated the general nature of fats and oils, and since his time a vast amount of detailed information has been obtained and many new forms made familiar. It was early suggested that rancidity is merely the development of some volatile acid, such as butyric, but investigation showed the untenability of this view. Recently, Wilmer C. Powick, of the U. S. Bureau of Animal Industry, has made an extensive research into the problem. His results are published in *Jour. Agric. Res.* (1923, **26**, 323). His conclusions are as follows:

(1) The odor of heptylic aldehyde ($C_7H_{14}O$) in itself and in association with fresh fats seems to justify Scala's view that this substance is fundamentally responsible for the characteristic rancid odor.

(2) A long list of acids, aldehydes and volatile derivatives from fats is given, all of which are excluded as having any concern with the condition. Nonylic aldehyde ($C_9H_{18}O$), however, may be partly responsible. Other substances, yet not isolated definitely, may take part in determining rancidity. Kreis' test—the use of a hydrochloric acid-phloroglucin mixture—was found not to be absolutely definite, as usually applied, but by a special spectroscopic method it may be made trustworthy. Non-rancid cottonseed oil will, for instance, give a color under the ordinary method of the test. Powick discusses at some length the chemistry of rancidity production, giving a very complicated view, which is mainly indicative of oxidation of oleic acid. No discussion is presented as to the determining influences, but this phase is really outside the scope of the investigation. The view generally entertained is that protein matters are responsible for fermentive influences, but this is mere speculation at present. H. L.

An Acoustical Probe. Z. CARRIÈRE. (*Jour. de Phys. et le Radium*, Nov., 1923.)—For exploring the state of the air vibrating in organ pipes or other cavities, a membrane strewn with sand has been a classical apparatus. In place of this the author uses a trapezoid of aluminum leaf supported by a piece of foil. The size of the leaf must correspond to the frequency of the mass of air to be investigated. For a pipe of 230 vibrations the very indefinite dimensions "10×4 mm.^{2}" are given, the thickness of the leaf being .03 mm. When such a probe is introduced into a vibrating mass of air it will in general be set in motion. Let its plane be perpendicular to the direction of motion of the air particles. Its amplitude will be a maximum at the loops and zero at the nodes. If there be a glass side and a microscope be focussed on the leaf, the position of the nodes can be sharply determined, since the displacement of the leaf from the node by even a little distance is followed by a noticeable motion. G. F. S.

RADIATION.*†

BY

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The correspondence between the two pictures is so close that often the same terminology may be employed for both. It is customary in X-ray work to name the various radiations which are characteristic of a given element "K," "L," "M," "N," etc. In terms of the static-atom the "K" rays are caused by the electrons returning to the inmost "pair" which is complete in helium; the "L" rays are due to the electrons returning to the inmost "octet" which is complete in neon; the "M" rays are due to the electrons returning to the "octet" which is complete in argon, etc. A painstaking search by Richtmeyer [48] failed to disclose any characteristic wave-lengths shorter than the "K." This means that there are no "J" electrons closer to the nucleus than the "K" electrons. This is consistent with the fact that the mass of chemical and physical-chemical evidence upon which the static atom is founded requires no such electrons. The various "shells" of electrons in the static-atom picture may, therefore, be called the "K," "L," "M," etc., shells. In terms of the more simple pictures of the Bohr atom, the "K" rays are due to electrons whose quantum number, n, decreases to 1. Such an electron has dropped into the "K ring" or "K orbit." When the electron decreases its quantum number to 2, it gives off "L" rays, and the electron is said to have dropped into the "L ring." This simple nomenclature is hardly sufficient for the more complicated picture to be described later, where it is more convenient to designate the action of the electrons entirely by their quantum numbers.

Now in deriving equation (63) for the Bohr atom we made the implicit assumption that the mass of the nucleus was infinite, so that it remained stationary while the electron moved about it. The corresponding assumption was made in equation (70) for the radiating static atom. It was for this reason that the Rydberg constant of equation (69) was called N_∞. In the actual atom,

* Presented at a meeting held Thursday, January 31, 1924.

† Continued from the April issue, p. 478.

the mass of the positive nucleus is, of course, finite, so that equation (63) becomes

$$\frac{Ze^2}{(\alpha+A)^2} = m\alpha\omega^2 = MA\omega^2 \tag{74}$$

where M and A are the mass and the radius of orbit, respectively, of the nucleus. The potential energy of (61) becomes

$$V = -\frac{Ze^2}{\alpha + A} \tag{75}$$

and the kinetic energy of (62) is

$$\begin{aligned} T &= \frac{1}{2} mv_1^2 + \frac{1}{2} Mv_2^2 \\ &= \frac{1}{2} m\alpha\omega^2 (\alpha + A) \\ &= \frac{1}{2} \frac{Ze^2}{\alpha + A}. \end{aligned} \tag{76}$$

Equation (66) is replaced by two equations.

$$\alpha = \frac{n^2h^2}{4\pi^2 mZe^2} \tag{77}$$

$$A = \frac{n^2h^2}{4\pi^2 MZe^2} \tag{78}$$

so that (67) becomes

$$\begin{aligned} U_j &= -\frac{2\pi^2 Z^2 e^4}{n_j^2 h^2 \left(\frac{1}{m} + \frac{1}{M}\right)} \\ &= -\frac{2\pi^2 Z^2 e^4}{n_j^2 h^2 \left(\frac{1}{\mu}\right)} \\ &= -\frac{2\pi^2 Z^2 e^4 \mu}{n_j^2 h^2} \\ U_k &= -\frac{2\pi^2 Z^2 e^4 \mu}{n_k^2 h^2}. \end{aligned} \tag{79}$$

This gives instead of (69)

$$\nu = \mu/m N_\infty Z^2 \left(\frac{1}{n_j^2} - \frac{1}{n_k^2}\right). \tag{80}$$

In terms of the static atom this means that the constant of equation (1) is no longer given by equation (70), but is given instead by

$$c = \frac{\frac{1}{m + M}\left(\frac{nh}{2\pi}\right)^2}{Ze^2}. \tag{81}$$

By means of equation (80) it is possible to calculate (1) the ratio of the mass of the hydrogen atom to that of the electron, and (2) the ratio e/m.

The true Rydberg constant for hydrogen is

(82)
$$N_H = \mu/_m N_\infty = \frac{M_H}{m + M_H} N_\infty = \frac{N_\infty}{1 + \frac{m}{M_H}}.$$

Similarly, the Rydberg constant for helium is

(83)
$$N_{He} = \frac{N_\infty}{1 + \frac{m}{3.968 M_H}}$$

because the ratio of M_{He} to M_H is as 4.00 is to 1.008. Dividing (82) by (83)

(84)
$$\frac{M_H}{m} = \frac{N_H - .252\, N_{He}}{N_{He} - N_H}.$$

Experimentally, the empirical equations for the spectra of hydrogen and helium are satisfied by

(85)
$$N_H = 109{,}677.691 \times 3 \times 10^{10}$$
$$N_{He} = 109{,}722.144 \times 3 \times 10^{10}$$
$$N_\infty = 109{,}737.11 \times 3 \times 10^{10}.$$

Substituting these values in (84), we have

(86)
$$\frac{M_H}{m} = 1845.3.$$

The mass H of the hydrogen *atom* (nucleus and one electron) is therefore 1846.3 times the mass of one electron.

Now,

(87)
$$\frac{H}{m} = \frac{e/m}{e/H}$$

e/H has been measured with great care by Vinal, of the Bureau of Standards,[40] and found to be 9650 absolute electromagnetic units per gram. From equation (87) we have

(88)
$$e/m = 1846.3 \times 9650 = 1.782 \times 10^7 \text{ als. el. mag. units/gram.}$$

The degree of accuracy of the experimental values of the various constants [40, 41] in equations (84) and (87) is such as to give an uncertainty of about 1 per cent. in the final value of e/m in equation (88). It is interesting to note that the value from equation (88) is almost exactly 1 per cent. higher than the accepted value of $e/m = 1.766 \times 10^7$.

Now equations (69) and (80) were based on the simplest possible form of orbit, the circle. But a circle is only a special case of the more general elliptical orbit. If the orbit is assumed to be an ellipse, equation (58) must be replaced by two equations, the first of which

$$\int p dq = n_a h \tag{89}$$

applies the quantum relation to the azimuth, and the second of which

$$\int p dq = n_r h \tag{90}$$

applies it to the radius. The n for the circle of equation (58) corresponds to the n_a of (89). It follows from (89) and (90) that if ε is the eccentricity of the ellipse

$$1 - \varepsilon = \frac{n_a^2}{(n_a + n_r)^2}. \tag{91}$$

This means that instead of the simple form of equation (67) we now have

$$U_j = -\frac{2\pi^2 Z^2 e^4 m}{h^2}\left(\frac{1}{n_{a_j} + n_{r_j}}\right)^2 \tag{92}$$

$$U_k = -\frac{2\pi^2 Z^2 e^4 m}{h^2}\left(\frac{1}{n_{a_k} + n_{r_k}}\right)^2$$

so that (80) becomes

$$\nu = \mu/m N_\infty Z^2\left[\frac{1}{(n_{a_j} + n_{r_j})^2} - \frac{1}{(n_{a_k} + n_{r_k})^2}\right] \tag{93}$$

Sommerfeld [42] has complicated this formula still more by taking account of the change in velocity of the electron, and, therefore, its change in mass as it goes around an elliptical orbit. In such a case, equation (92) takes the form

$$U = -\frac{2\pi^2 Z^2 e^4 m}{h^2}\left(\frac{1}{n_a + n_r}\right)^2\left[1 + \frac{\alpha^2 Z^2}{(n_a + n_r)^2}\left(\frac{1}{4} + \frac{n_r}{n_a}\right) + \cdots\cdots\right]. \tag{94}$$

The two terms in the bracket are really only the first two of a convergent series, but $\alpha = \frac{2\pi e^2}{hc} = 7.27 \times 10^{-3}$ is so small that the other terms may be neglected at this point. Later in deriving equation (100) we will use the first five terms of the series.

Equation (93), therefore, becomes

$$\nu = \mu/m \, N_\infty Z^2 [J - K] \tag{95}$$

where

$$J = \frac{1}{n_{a_j} + n_{r_j}} + \frac{\alpha^2 Z^2}{(n_{a_j} + n_{r_j})^4}\left(\frac{1}{4} + \frac{n_{r_j}}{n_{a_j}}\right) + \cdots\cdots$$

and

$$K = \frac{1}{n_{a_k} + n_{r_k}} + \frac{\alpha^2 Z^2}{(n_{a_k} + n_{r_k})^4}\left(\frac{1}{4} + \frac{n_{r_k}}{n_{a_k}}\right) + \cdots\cdots .$$

Even all this is consistent with the picture of the static atom if the "constant" of Thomson's law of force (equation (1)) is

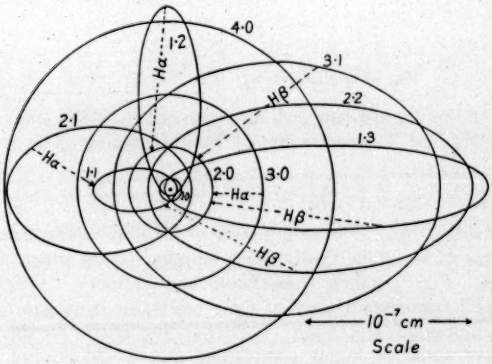
$$c = \frac{\frac{1}{m} + \frac{1}{M}}{Ze^3}\left(\frac{h^2}{4\pi^2}\right)\left[(n_a + n_r)^2 - \alpha^2 Z^2\left(\frac{1}{4} + \frac{n_r}{n_a}\right) + \cdots\cdots\right] \tag{96}$$

Equation (95) gives us a picture of an atom whose electrons, in terms of the Bohr theory, have elliptical orbits which slowly precess. Fig. 3, taken from Foote and Mohler's "Origin of Spectra," represents a view of these orbits for hydrogen at any one instant in this precession. Corresponding figures for the other elements would be highly complicated and would require three dimensions. If we interpret the equation in terms of a static atom, it probably means that the "electron shells" revolve about the nucleus, or, what is the same thing from the standpoint of relative motion, that the nucleus revolves with respect to fixed "shells" of electrons.

It should be noted that this picture of a rotating atom only applies to atoms in the gaseous state. There are no experimental facts which require the rotation of atoms in solids and, indeed, all the facts of crystal structure indicate that the atoms of solids do not rotate. We have seen in the discussion of the static atom that K^+ and Cl^- pack in the crystals of the corresponding alkali halides like cubes with rounded corners. It may be shown [11] that the shape and size of these ions are very approximately the same. If the electron shells, or their equivalent in the Bohr atom,

revolved at random, we should expect these ions to pack in the crystal of KCl in the closest packing of equal numbers of oppositely charged spheres of equal size, that is, in a body-centred cubic structure like the equal "spherical" ions of Cs^+ and I^- in CsI. Actually K^+ and Cl^- do not pack in such a close packing, but instead, take a simple cubic structure of ions. This would seem to indicate that there is some constant directive force between the positive and negative ions, and therefore no random rotation. It may very well be that the mechanical constraints so

FIG. 3.



Bohr orbits in an atom of hydrogen.

imposed contribute to the enormous number of possible frequencies of the Planck "oscillators" mentioned in the discussion of black-body radiation.

Equation (95) should be further corrected by altering the value given to Z to account for the shielding effect on the nucleus of the electrons near it. The new value is

$$Z' = Z - S \tag{97}$$

where S is called the "nuclear defect." The value assigned to S depends, of course, upon the picture which is assumed for the orbits of the various electrons. Values may be found in the literature for S on the assumption (clearly untenable) that all the electrons lie in concentric co-planar orbits. Other values may

be found based on the assumption that half the electrons lie in concentric co-planar orbits above the nucleus, and the other half in similar orbits below the nucleus. This second assumption is a close approximation to the static-atom picture of the simpler elements. Sommerfeld finds [42] empirically that the nuclear defect of the K and L rings for all atoms of more complex structure, *i.e.*, from atomic number 40 to 92, is approximately 3.63. This is often given in round numbers 3.50. With large atomic numbers the percentage error involved is negligible. This must mean that for these atoms, at least, the structure of the innermost shells is nearly independent of the atomic number.

The final form of the equation for the frequencies which can be emitted by a single atom of a gas is, therefore,

$$\nu = \mu/m N_\infty (Z-S)^2 (J-K) \tag{98}$$

where K and J have the same significance as in equation (95).

It follows directly from equation (91) that

$$\frac{b}{a} = \frac{n_a}{n_a + n_r} \tag{99}$$

where $2a$ and $2b$ are the major and minor axes, respectively, of orbits corresponding to some definite amount of energy; n_r may be any positive integer including zero. When n_r is zero the orbit is circular; otherwise it is elliptical. n_a cannot be zero, for the orbit would then be a straight line passing through the nucleus, but it may be any positive integer except zero.

On the basis of equation (80) the hydrogen line H_α corresponds to $n_j = 2$ and $n_k = 3$. On the basis of equation (93) $n_{a_j} + n_{r_j} = 2$ and $n_{a_k} + n_{r_k} = 3$. This would mean that $n_{a_k} + n_{r_k}$ may be $(1+2)$, $(2+1)$, or $(3+0)$, and $n_{a_j} + n_{r_j}$ may be $(1+1)$ or $(2+0)$. There are therefore six ways of getting the H_α line. Equations (95) and (98) show that these six ways should correspond to six slightly different energies and therefore six slightly different frequencies. This is because in the derivation of (95) the change of mass of the electron at different parts of the orbit is taken into account. These six frequencies are reduced to three by the "principle of selection." [43] In our formulas we have assumed merely that energy lost by the electron appears as energy in the radiation. In other words, we have applied the ordinary law of conservation of energy to our picture of the mechanism of

radiation. If, however, we assume in addition that we must apply the law of conservation of moment of momentum (at least in the absence of an external electric or magnetic field), then n_a may change only by +1 or −1. This principle enables us to "select" (therefore the name) from the six frequencies mentioned above those which are physically possible. We may, therefore, calculate at once the frequencies or the wave-lengths of the lines into which H_α can be resolved. The results of such calculations are given in Table X.

TABLE X.

Fine Structure of the H_α Line.

ν	λ	n_{a_j}	n_{r_j}	n_{a_k}	n_{r_k}	$n_{a_j} n_{r_k}$	
45,700.353 × 10^{10}	6564.501	3	0	1	1	−2	Not possible
45,700.245	6565.516	2	1	1	1	−1	Possible
45,699.921	6564.563	1	2	1	1	0	Not possible
45,699.258	6564.658	3	0	2	0	−1	Possible
45,699.150	6564.674	2	1	2	0	0	Not possible
45,698.223	6564.720	1	2	2	0	+1	Possible

Now because of the Doppler effect, the width of a line in the hydrogen spectrum cannot be reduced below .051 Å. This means that it should be impossible to separate the two lines $\lambda = 6564.658$ and 6564.720 with slits of finite width. A separation should be possible between $\lambda = 6564.658$ and $\lambda = 6564.516$, whose difference in wave-length is .142 Å. or $\Delta\nu_H = .987 \times 10^{10}$. If we had calculated $\Delta\nu_H$ directly from equation (95) we would have found $\Delta\nu_H = 1.093 \times 10^{10}$; because, by not applying the principle of selection, we should have considered the doublet to be the first and fourth, second and fifth, or the third and sixth of Table X. The mean of the experimental of values of four observers [44, 45, 46, 47] is 1.005×10^{10}, which agrees well with both of the calculated values 1.093×10^{10} and $.987 \times 10^{10}$ given above. Similarly the fine line structure of H_β may be calculated. The orbits involved in these calculations are marked in Fig. 3. The corresponding fine structure of the line $\lambda = 4686$ of ionized He may be calculated to give five lines, four of which may be resolved. Here again the calculated frequencies agree with the experimental value to within the experimental error. But a still more striking confirmation of equation (98) may be obtained in the case of the

L doublet of the characteristic X-ray spectrum of uranium. Now in the L lines, the total quantum number is 2, so that $n_a + n_r$ must be $(1+1)$ or $(2+0)$. For this case, equation (98) gives

$$\Delta\nu = \nu_1 - \nu_2 \tag{100}$$
$$= \mu/m N_\infty \frac{\alpha^2}{16}(Z-S)^4\left[1 + \frac{5}{8}\alpha^2(Z-S)^2 + \frac{53}{128}\alpha^4(Z-S)^4 + \frac{831}{2048}\alpha^6(Z-S)^6 + \cdots\right].$$

Now the mass of the uranium nucleus is so great in comparison with the mass of an electron that we may call $\mu/m = 1$.

$N_\infty = 329{,}211.33$ (see equation (85))
$\alpha = 7.27 \times 10^{-3}$ (see equation (94))
$Z = 92.0$ (atomic number of uranium)
$S = 3.63$ (see equation (97)).

Substituting these values in (100), we get

$$\Delta\nu_U = (6.70 + 1.74 + .48 + .19 + \ldots) \times 10^{17} \tag{101}$$
$$= 9.11 \times 10^{17}.$$

The experimental value is 9.06×10^{17}.

Every value except S entering into the calculation of (101) is the result of experiments which have nothing to do with the X-ray spectrum of uranium. The value of S is entirely empirical, but at least it is not a value whose application is limited to the case of uranium alone. The same value may be used in corresponding calculations for the L spectra of all elements for which Z is equal to or greater than 40. For lower atomic numbers S becomes smaller until for hydrogen $S = 0$. Equation (98) therefore enables us to calculate, to within the experimental error, the spacings of the "fine structure" of spectral lines for the whole series of chemical elements. In terms of the frequency differences, $\Delta\nu$, this gives a range of ninety million fold. Its exactness and range give an unusual feeling of confidence in the type of picture which gave rise to it.

In accordance with this picture, a single atom of a gas is composed of a central positive nucleus surrounded by electrons which are able to exist only in definite states so that they possess energy only in definite amounts. These amounts of energy may be calculated on the basis of definite, precessing, elliptical, orbits which have one focus at the positive nucleus. When one of these electrons changes from a state (orbit) of one energy content to

one of a lower energy content, it gives off the difference in the form of radiation. This transfer of energy obeys the laws of conservation of energy and conservation of moment of momentum. The mechanism of the transfer is such that the frequency of the radiation is directly proportional to the amount of energy available. The spectra of single atoms of gases are, therefore, composed of groups of *lines*.

In a gas whose atoms are combined together to form molecules, the constraints on the electrons are more complicated so that their "definite states" are not as simple as in the case of a monatomic gas. Band spectra are the result. In a liquid or solid, "definite states" are so complex that practically any change in energy content is possible up to the limit imposed by the temperature itself. We therefore have "black-body" radiation, or an approximation to it, from liquids and solids. The approximation is closer the higher the temperature.

If the production of light is caused by the vibration of electrons within an atom, it is reasonable to assume that the frequencies of the light would be altered by the presence of an electric or a magnetic field. This is confirmed experimentally by the Stark and the Zeeman effects. An electric field also gives rise to an apparent violation of the "principle of selection." For instance, in the case of ionized helium, two lines ($\lambda = 4685.53$ and $\lambda = 4684.02$) are observed experimentally which are mathematically possible from equation (98), but which would be ruled out by the "principle of selection" if there were no electrostatic field.

(B) THE EFFECT OF MATERIAL SUBSTANCES IN THE PATH OF RADIATION.

So far, we have considered the laws in accordance with which a substance (the atom) may produce radiation. There remains to be considered the interaction between this radiation and material substances in its path. This will include the ordinary phenomena of the photo-electric effect, photographic effect, fluorescence, reflection, scattering, refraction, polarization and interference. The quantum theory and the photo-electric effect have been held by many people to favor some sort of a corpuscular theory, while polarization and interference have long been held to favor some sort of wave theory. By a consideration of these subjects, we may hope to eliminate certain ideas which might otherwise appear

plausible, and in this way we may arrive at some definite ideas as to the nature of radiation.

Photo-electric Effect.—It is well known that if light falls upon certain metallic surfaces, electrons tend to leave the surface. If the clean surface is made the negatively charged electrode of a vacuum tube, the number of electrons leaving the surface per second is proportional to the intensity of the incident light. Now let the polarity be reversed. If the field is weak, only the slowest moving electrons fail to emerge from the surface. As the field is increased, a potential is at last reached such that not even the fastest electrons leave the illuminated surface. It is found by experiment that the maximum speed, as measured in this way, does not depend in any way on the intensity of the incident light, but does depend upon its frequency. If the experiment is repeated with various frequencies of light, it is found that the energy of the fastest electrons is represented by the Einstein equation.[34]

$$E = h\nu - \omega \tag{102}$$

where h is the same constant as in Planck's equation, ν is frequency of the incident light and ω is a constant for a given surface-material. As far as equation (102) is concerned, $h\nu$ either represents the energy in the incident "light wave" or it is the energy amount of energy which an electron can pick up from a light wave of frequency ν. ω is the energy required to separate the electron from the surface; it is this value that was calculated for the alkali metals in Table VI.

Now equation (102) leads to some rather far-reaching conclusions. The photo-electron must acquire its energy from the incident light. If the incident light represents an energy of one erg per square centimetre (which is the energy received per second per square centimetre from a standard candle at a distance of 2 metres), and if the area of cross-section of an atom is taken to be 10^{-16} square centimetres (as determined by X-ray diffraction methods on atoms in crystals), and if the energy of the photo-electron is 10^{-12} ergs (it can actually be greater than this for 10^{-12} ergs only corresponds to 0.6 volt on the electron) then the electron should, on any ordinary wave theory, require 10^4 seconds or about two and three-quarters hours before it could pick up its total energy from the light wave. Experimentally there is no

measurable time interval between the arrival of the incident light and the emission of the photo-electron.

The situation has been visualized by W. H. Bragg thus. Suppose two ships are a thousand miles apart on the ocean, and suppose that a plank A is floating on the water near one of the ships. Now let an exactly similar plank B be dropped from the deck of the other ship into the water. A thousand miles away, plank A suddenly jumps from the water to the height of the ship's deck! It is quite evident that no ordinary wave spreading out in all directions from the plank B could accomplish this. It would require a wave of narrow wave front travelling directly from B to A.

This illustration makes it appear that the $h\nu$ of equation (102) must represent the total energy of the wave which is responsible for the photo-electric effect. Apparently then, an electron in the source of light lost an amount of energy E_0 which appeared as light of a frequency such that $E_0 = h\nu$. The whole of this "quantum" reaches a single electron in the sensitive surface of the photo-electric cell and appears as kinetic energy of that electron. Part of this energy, ω, is used up in sending the electron past the surface, leaving an energy $E = h\nu - \omega$ in the photo-electron.

It may be a little premature to say definitely that the action of a photographic emulsion is a special case of the photo-electric effect. But at any rate it has been shown[49] that the "grains" in the emulsion are not all affected at once. Even a minute exposure to light appears to make some grains developable without in any way affecting neighboring grains. Apparently no appreciable time is required for affecting those grains which become developable. Further exposure to light seems to result primarily in causing other grains, which had hitherto been unaffected, to become developable. Silberstein has likened the effect to that of a "rain of light" rather than a "flood of light." This action of a photographic plate and the photo-electric effect together form the experimental basis which justifies the effort to find some sort of corpuscular theory of light which is consistent with the other known facts of radiation.

The foregoing is sufficient to illustrate the difficulty in thinking of light as a system of concentric spherical wave fronts. It is quite true, as was stated in the discussion of Planck's equations, that the quantum theory had its origin in the probability that a

finite increment of energy could be radiated from an "oscillator." But the action of the photo-electrons tempts us to go further and picture the light wave itself as a "shaft of light" carrying a "quantum" of energy $h\nu$. If such shafts were given off from the light-source now in one direction, and now in another, the average effect would be that of a spherical wave whose intensity varied inversely as the square of the distance from its source, and at the same time the photo-electric effect could be easily explained. Such a picture has been employed by Silberstein in his theory of "darts." It is implicitly contained in the recent quantum theory of diffraction and of scattering. We shall return to this picture later after we have discussed some other consequences of the quantum theory.

Fluorescence.—The quantum theory offers a remarkably simple explanation of the experimental fact, known as Stokes' law, that the frequency of light which causes fluorescence must be slightly greater than that of the fluorescent light itself. The energy of the incident light depends upon its frequency so that $E_1 = h\nu_1$. The energy of the fluorescent light is $E_2 = h\nu_2$. Since the process cannot be 100 per cent. efficient, E_2 must be less than E_1, so that ν_2 must necessarily be less than ν_1. An idea of the efficiency of the fluorescent process may be had from the following. The critical absorption frequency of a substance for X-rays is about a quarter of a per cent. greater than the greatest corresponding X-ray emission frequency for the same substance.[35] It is this shortest emission line which must be excited if the substance is to give off its X-ray fluorescence spectrum. Since E_1 and E_2 vary directly as ν_1 and ν_2, the energy loss for this line directly due to the act of fluorescence alone is a quarter of a per cent. This means, then, that the efficiency of producing fluorescence is 99¾ per cent. Of course, this does not take into account losses not directly connected with the act of fluorescence, such as scattering of the incident beam in the fluorescing substance and "opacity" of the substance to the fluorescent X-rays.

General X-radiation.—If we subtract from the spectrum given off by an X-ray tube those X-rays which are characteristic of the anti-cathode, the remainder of the spectrum is a continuous spectrum which is called "general" or "white" X-radiation. If the intensity of this general radiation is plotted against wavelength, a graph is obtained which is of the same general form as

that for black-body radiation. On the long wave-length end it approaches the axis of zero intensity asymtotically. On the short wave-length end it cuts the axis of zero intensity abruptly.

Now the maximum energy which an electron can give up on hitting the target is

$$E = (P.D)e.$$

The highest frequency of X-rays which can be given off is therefore given by

$$(P.D)\,e = h\nu. \tag{103}$$

Equation (103) was experimentally confirmed by Duane and Hunt [36] and later by others.[37] These measurements form one of the means of obtaining precise values of h.

Absorption and Transmission.—If radiation is to be considered in terms of a corpuscular theory at all, it must consist of a periodic succession of corpuscles. In the case of the simplest sort of periodicity, the number of corpuscles passing a given point per second would be the "frequency" of the light, and the distance from one corpuscle to the next would be the "wave-length." In order that material corpuscles may be able to pass through a transparent substance, it is necessary that they be very small in cross-section, at least as small as the electron. If radiation is, instead, a wave motion, the phenomena of transmission impose no requirements on the area of the wave front.

No material substance is equally transparent to radiation of all frequencies. This must mean that transparency to radiation depends upon the damped natural frequencies of the electrons in the atoms of the transparent substance. Lorenz has suggested that the effect of a frictional damping force could be produced by a collision between molecules which contain vibrating electrons with other molecules. We shall see later that the lost energy can be easily accounted for in a quite different way in terms of the recoil of electrons.

In either case we may couple this lost energy with a hint given by the quantum theory. Experimentally, the critical absorption frequency is slightly greater than the natural frequency of vibration. The energy absorbed is $E_1 = h\nu_1$. The slight loss E in energy means that the frequency reëmitted $= \nu_2 = \frac{E_1 - E}{h}$. In other words, a critical absorption must always be accompanied

by fluorescence at a slightly lower frequency. Therefore, if the frequency of the radiation is equal to, or slightly greater than, the natural frequency of some electron in the atom, then that electron should absorb energy from the light. The atom of which that electron is a part is, therefore, "opaque" to that frequency of radiation. Now we know of no way by which an electron may be influenced except by (1) an electrostatic field, (2) a magnetic field, or (3) possibly by the impact of a material particle, as, for instance, might be assumed in the passage of electrons along a metallic bar, one end of which is heated. This means that if radiation is a wave motion, it must be a wave motion of an electrostatic field or magnetic field. Since we cannot experimentally produce one without the other, any wave theory of radiation must necessarily be an electromagnetic wave theory.

In terms of a corpuscular theory, it would be possible to affect an electron by the passage of either a magnetic corpuscle or an electrically charged corpuscle. The corpuscle would not have to impinge directly upon the electron, it would be sufficient for it to pass near it. But such a corpuscular theory is quite untenable for the direction of the corpuscles should be affected by strong electric or magnetic fields. Experimentally such an effect has never been found. Moreover, it is easy to show that the corpuscle cannot be both non-magnetic and electrically neutral, for in such a case its passage through space could only be stopped by direct impact upon an electron or an atomic nucleus. Now, in order that a corpuscle may pass through a "transparent" substance it must be small enough to pass between, or through, the atoms of that substance. Since many such substances are amorphous (for instance, glass shows no regularity of atomic arrangement when examined by X-ray methods), they offer no large free passageways for the corpuscles. The corpuscles would, therefore, have to be of a size at least as small as that of an electron. But we know that single crystals of Fe, Mo and W are quite opaque to ordinary light, in spite of the fact that their crystal structure is such (body-centred cubic) as to provide passageways in three directions clear through the crystals. These passageways are so large as to take up half the area of a cube-face of the crystal, so that when properly oriented, even a thick crystal could only cut off half of a neutral corpuscular radiation. This is so contrary to the well-known facts of the opacity of these metals that the neutral-corpuscle

idea is as untenable as the magnetic or electrostatically charged corpuscle.

There remains, then, only some sort of an electromagnetic wave theory. According to such a theory, radiation consists of trains of waves, each wave consisting of a "displacement" (*i.e.*, changed condition) of an electrostatic field and of an electromagnetic field. These "displacements" are perpendicular to each other and to the direction of propagation of the wave. Two forms of this theory are to be considered, one in which the wave front extends continuously in a spherical shell around its source, the other in which the wave front is of very limited area. The first represents a wave front of constantly decreasing amplitude. The second is a highly directed beam (or "dart") of radiant energy such as might be imagined from an infinitesimal search-light.[49] According to this version of the electromagnetic theory, there are so many of these darts, and each covers such a small area with its little wave front, that they appear to most of our physical apparatus like continuous spreading waves. The inverse square law would still hold, for the number of darts cutting a unit area of an imaginary spherical shell would depend upon the inverse square of the distance. Only when the receiving mechanism is of such small dimensions as to be comparable to the area of cross-section of the tiny beam could the individual beams be distinguished. It has already been shown that the electrons in the sensitive surface of a photo-electric cell may be regarded as such receiving mechanisms. It is possible to regard the electrons in the sensitive emulsion of a photographic plate in the same way.[49]

It is probable that because of the ease with which it explains the photo-electric effect, this second form of the electromagnetic theory would have entirely displaced the spreading-wave form long ago if it had not been for certain difficulties in regard to diffraction which will be mentioned later.

Reflection.—The experimental facts of reflection are consistent with either type of electromagnetic wave theory. A good electrical conductor should be a good reflector of light. When such a conductor is placed in the alternating electrostatic field of a light wave, induced charges are caused which moment by moment tend to neutralize the electric component of the wave. In the same way the alternating magnetic field produces induced currents which tend to oppose the change in the field which produces them.

The wave cannot, therefore, penetrate the conductor. It must be reflected or absorbed. If a conductor could be found with zero resistance, all the energy of the incident wave would appear in the reflected wave. Actually there is some energy absorbed by the electrical resistance of the reflector. The reflectivity of a mirror surface is, therefore, in general, proportional to its electrical conductivity.

Non-conductors differ from conductors in that their electrons are in general not free to move in response to an electric or magnetic field. Such movement can only occur readily when the frequency of the field coincides with the natural frequency of one or more of the electrons or ions in the atom. Non-conductors should therefore show selective reflection. For instance, quartz shows selective reflection in the infra-red at $\lambda = 8.85 \times 10^{-4}$ cm. and $\lambda = 20.75 \times 10^{-4}$ cm. These frequencies may be calculated approximately on the basis of a silicon ion of charge 4, and oxygen ions of charge 2. It should be mentioned, however, that one would not assume from the crystal structure of quartz [50] that these elements were present as ions, for the quartz crystal is of the molecular, not of the ionic type. This may mean that the component atoms of even the molecular "valence compounds" are negatively charged. Quartz also shows selective reflection in the ultra-violet in the neighborhood of $\lambda = .10 \times 10^{-4}$ cm. due to the natural frequency of some of the electrons. Not even the metals are free from selective reflection, presumably because of the natural frequencies of electrons in their outmost complete shells. An example of this is the well-known reddish color of copper.

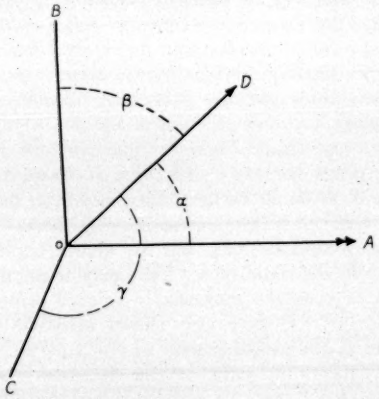
No substance is known which acts like a mirror, in the ordinary sense of the word, for X-rays. This is because the frequency of the radiation is so high that even in metals like copper and silver the "valence" electrons, which ordinarily make these metals conductors, cannot easily respond to such high frequencies.

Polarization.—It is considered one of the strong arguments in favor of the spreading electromagnetic wave theory that it explains polarization. If radiation were composed of material particles, polarization would be almost unthinkable, but if we assume a "dart" theory we can retain whatever advantages there may be in a corpuscular theory and at the same time explain polarization

because each "dart" is merely a train of electromagnetic waves of small cross-section.

Scattering.—When X-rays strike an object, that object acts like a source of X-rays, for X-rays which are nearly identical with the rays in the incident beam come out from the object in all directions. This effect may be interpreted either in terms of the spreading electromagnetic wave theory, or in terms of the dart-

FIG. 4.



of-electromagnetic-radiation theory. We will, therefore, briefly take up each of these viewpoints.

The classical spreading-wave theory assumes that the electrons of an atom can take up energy from a wave in any amount. Therefore, when an X-ray passes over an electron, the electron should be accelerated in the direction of the electric component of the wave. In the simplest form of this theory [51] the electrons are assumed to be of negligible size, and each electron is supposed to act independently of every other electron. With such a simple picture, the law of scattering may be derived in terms of a single electron. Let an X-ray (electromagnetic wave) travel in the direction *OA* (Fig. 4). Let the electric vector be in the direction of *OB* and the magnetic vector be in the direction of *OC*. Then an electron at *O* will, in accordance with the theory, be accelerated in the direction

OB. As a result it will radiate in all directions with an intensity proportional to $\sin^2 \beta$ where β is the angle between OB and the direction OD in which the scattered rays are measured. It is desired to express this intensity in terms of the angle α between the direction OD and the direction OA, in which the primary X-ray beam is travelling. Now

$$\cos^2\beta = 1 - \cos^2\alpha - \cos^2\gamma.$$

If we assume that the primary beam is impolarized, then *on the average*, $\cos^2 \beta = \cos^2 \gamma$, so that on the average

$$\cos^2\beta = \frac{1}{2}(1 - \cos^2\alpha).$$

Therefore

$$\sin^2\beta \quad (= 1 - \cos^2\beta) \quad = \frac{1}{2}(1 + \cos^2\alpha).$$

Therefore, if I_o is the intensity of X-rays scattered by a single electron in the direction of the primary beam, then the intensity I_θ at any angle θ is, on the average,

$$I_\theta = \frac{1}{2}(1 + \cos^2\theta)\, I_o.$$

In terms of the intensity, I, of the primary beam,

$$I_o = I \frac{e^4}{m^2R^2c^4}$$

where R is the distance along OD of Fig. 4 from the electron at o to the place where the intensity is measured, and where e, m and c have the ordinary meaning. If there are n atoms per unit volume, and if each atom has Z electrons outside the nucleus, then the intensity of X-rays scattered per unit volume is

$$I_s = nZI_\theta = nZI \frac{e^4}{2m^2R^2c^4}(1 + \cos^2\theta). \tag{104}$$

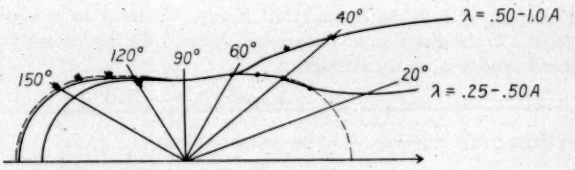
This would indicate that the graph of intensity of scattered X-rays at various angles should be symmetrical with respect to a plane drawn perpendicular to the direction of the primary beam at the place where the scattering substance is situated. This is shown by the dotted line in Fig. 5, which is taken from the *Bulletin of the National Research Council* (vol. 4, No. 20) by A. H. Compton. The graph shows that the equation fits the experimental facts very well if θ is 60° or more. As θ approaches zero, so that the direction is more nearly that of the primary beam, the scattered rays of long wave-length show experimentally a greater intensity than that given by the equation.

This is explained by assuming that the electrons in the atom are far enough apart so that there is an appreciable phase difference between the scattered rays given off by the various electrons. Taking account of this phase difference Debye [52] finds that if the electrons have fixed positions in the atom, equation (104) must be multiplied by the factor

$$\sum_m \sum_n \frac{\sin\left(\frac{4\pi S_{mn}}{\lambda}\sin\theta/2\right)}{\frac{4\pi S_{mn}}{\lambda}\sin\theta/2} \tag{105}$$

where S_{mn} is the distance from the mth electron to the nth electron. The summations indicated require that some particular atomic

FIG. 5.



× Scattering of soft X-rays (λ=.50 to 1.0 Å.). ● Scattering of moderately hard X-rays (λ=.25 to .50 Å.). Data from Owen. Dotted curve from equation (104). Solid curves from equation (106).

model be chosen. A. H. Compton,[53] assuming a random distribution of electrons on the surfaces of shells of radius ρ_m, ρ_n, etc., obtains a correction factor for equation (104) of

$$Z + 2\sum_1^{Z/2}\left(\frac{\sin 2k}{2k} - 2\frac{\sin^2 k}{k^2}\right) + 4\left(\sum_1^{Z/2}\frac{\sin k}{k}\right)^2 \tag{106}$$

where

$$k = \frac{4\pi\rho}{\lambda}\sin\theta/2.$$

The two corrections lead to nearly identical results. The full lines of Fig. 5 show how nearly (106) makes (104) fit the experimental facts [54] for X-rays of $\lambda = .50 - 1.0$ Å. and $\lambda = .25 - .50$ Å. In this range of wave-lengths the agreement is seen to be very good. But when very short X-rays or γ-rays are used the formula fails. It may be brought into approximate agreement with the experimental facts by assuming that the radius of the electron is

so large that interference occurs between the rays scattered from different parts of the same electron. This means that the radius of the electron must be assumed to be of the order of 4×10^{-10} cm. The result is that equation (104) must be further multiplied by one of the three following expressions:

(107) $\frac{\sin^2 x}{x^2}$ for a spherical-shell electron.

(108) $\frac{1}{x} \sum_0^\infty J_{2n+1}(2x)$ for a flexible-ring electron.

(109) $\left(1 - \frac{x^2}{3 \cdot 3!} + \frac{x^4}{5 \cdot 5!} - \cdots\right)^2$ for a non-rigid spherical electron.

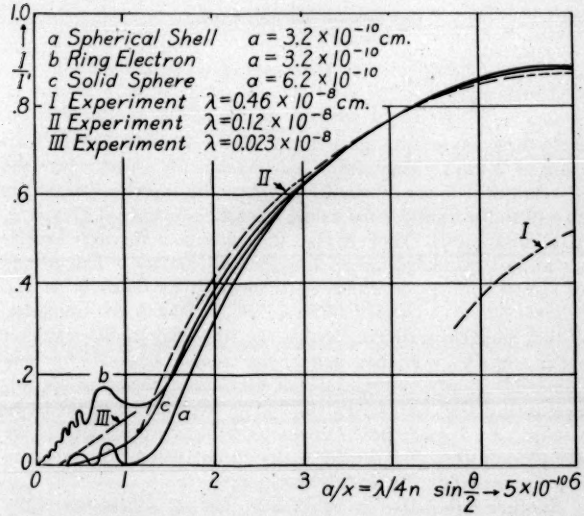
where $x = \frac{4\pi a}{\lambda} \sin \theta/2$ and where a is the radius of the electron. These three expressions give almost identical results for the scattering of X-rays. They differ considerably for γ-rays where the wave-length is much shorter. For γ-rays the experimental results fall within the limits of the values calculated by the aid of (107), (108) and (109). For X-rays the agreement between experimental and calculated values is quite satisfactory. The results are shown in Fig. 6, which is taken from the *Bulletin of the National Research Council* (Vol. 4, No. 20,) by A. H. Compton.

It is found experimentally that the frequency of the scattered rays is slightly lower than that of the incident beam.[55, 56] This is difficult to explain on the basis of the classical theory of spreading waves. The nearest approach to an explanation seems to be in terms of a hypothetical J characteristic radiation. But as we have already seen,[48] a careful experimental search has failed to show the existence of any such radiation.

Another alternative is to assume that all substances really fluoresce when giving out scattered radiation.[53] This would, of course, be a peculiar kind of fluorescence. It would not be limited to a particular narrow range of frequencies as in the ordinary case where fluorescence is caused by a natural frequency of one of the electrons in an atom, or by the natural frequency of a group of electrons in an atom or molecule. We have already seen that the quantum theory requires that the fluorescent radiation must be of lower frequency than the primary beam. From this viewpoint the suggestion of a "general fluorescence" really is a way of saying that the efficiency of the scattering process is less than 100 per cent. The spreading-wave theory considers the

scattering electron to be moved in the direction of the electric vector of the wave. This means that the electron must acquire energy which is later given out in the form of scattered radiation. Now, if this energy in the direction of the electric vector does not represent the whole quantum $h\nu$ of the primary wave, then the frequency of the scattered rays must be less than the frequency

FIG. 6.



Scattering of hard X-rays and gamma rays. Broken lines give Compton's experimental data. Full lines give values calculated from equations (107), (108) and (109).

of the primary beam. The experimental data are, therefore, accounted for qualitatively. It remains to find out what happens to that portion of the energy which does not appear as motion of the electron in the direction of the electric vector. This problem has been attacked recently by A. H. Compton,[55] as follows:

Since an electromagnetic wave-train represents an energy, and has a definite velocity, c, it may be said (at least figuratively) to have mass. Now in equations (61) and (64) it was found that the potential energy and the kinetic energy of an electron are equal

in a Bohr orbit. The total energy of the electron in any orbit is, therefore, $2(\frac{1}{2}mv^2) - mv^2$. It is the foundation principle of modern quantum theory that the energy of a wave-train of radiant energy is equal to the total loss in energy of an electron in dropping from one orbit to another, so that

$$h\nu = mv_k^2 - mv_j^2 = m\bar{v}^2 = \underline{m}\,c^2 \tag{110}$$

where $\underline{m}$ is the equivalent "mass" of the radiation and c is the velocity of light. The corresponding momentum is

$$\underline{m}\,c = \frac{h\nu}{c}. \tag{111}$$

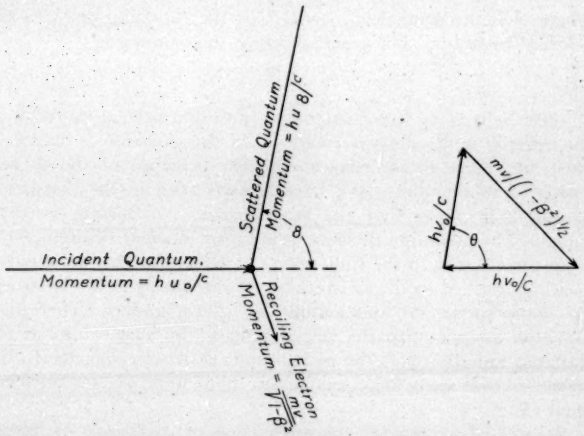
If a light wave has momentum, it should produce a recoil of the electron with which it collides. If this collision is elastic, it must take place in accordance with the principle of the conservation of momentum. We have already seen in the discussion of the Bohr atom that the fine structure of spectra may be explained by assuming the conservation of moment of momentum from the electron to the radiation. If we accept this assumption as being justified by the end-results, we can hardly refuse to accept the conservation of momentum as given above. Therefore, Compton assumes that the vector sum of the momentum of the scattered radiation plus the momentum of the electron due to the impact of the wave must equal the momentum of the incident beam (Fig. 7).

It is hard to imagine the application of a triangle of forces to a spherical spreading wave, but easy to imagine it in the case of a "dart" of light if only the cross-section of the "dart" is small enough. In fact, if the cross-section of the "dart" is comparable with that of an electron,[49, 57] and if we consider that each wave in this "dart" acts as if it had mass and momentum, then there is hardly any difference between the dart-of-electromagnetic-radiation theory and a corpuscular theory. In fact, such a viewpoint is sometimes briefly called a corpuscular theory [57] in the literature. The regular succession of waves in the dart corresponds to a regular succession of corpuscles of a corpuscular theory. The objections raised to a theory of electrically charged or magnetized corpuscles in the discussion of absorption are removed, for the dart would be unaffected by passing through

an electrostatic or a magnetic field, and yet it would have both electric and magnetic characteristics.

If we accept Silberstein's estimate [49] of the radius of a "dart" of blue light as $.176 \times 10^{-4}$ cm. and further assume that the radius is smaller with smaller wave-lengths, we avoid the difficulty encountered with the corpuscular theory in explaining the opacity of body-centred cubic crystals like iron and tungsten. Since the

FIG. 7.



Impact of a dart of small cross-section on an electron.

diameters of the passageways in these crystals are of the order of 1.5×10^{-8} cm., "darts" of visible light could not pass through, so that these metals should be opaque to ordinary light even in a direction perpendicular to the cube-face. It would be interesting to measure the opacity to X-rays of a thin film of a body-centred cubic substance cut parallel to the cube-face. The experimental difficulties would be enormous because the angle subtended by these passageways is very minute even when thin films are used.

By assuming that the triangle of forces of Fig. 7 may be applied to the scattering of radiation, Compton implicitly assumes either the electromagnetic-dart theory or some other sort of a "corpuscular" theory. Since Compton's calculations apply to the

scattering of X-rays, the cross-sectional area of the "dart" is probably small enough so that the whole "dart" may be considered as colliding with the electron. If the primary beam has a frequency ν_o and if the scattered beam has a frequency ν_θ, then

$$\Delta\nu_o = \nu_o - \nu_\theta$$

and the energy equation is

$$h\nu_o = h(\nu_o - \Delta\nu_o) + \frac{1}{2}mv^2 \tag{112}$$

where m and v refer to the mass and recoil-velocity of the electron. The corresponding momentum equation is

$$(mv)^2 = \left(\frac{h\nu_o}{c}\right)^2 + \left(\frac{h(\nu_o - \Delta\nu_o)}{c}\right)^2 - 2\left(\frac{h\nu_o}{c}\right)\left(\frac{h(\nu_o - \Delta\nu_o)}{c}\right)\cos\theta\,. \tag{113}$$ *

Substituting the value of v^2 from (113) into (112) and neglecting $(\Delta\nu_o)^2$, we have

$$\Delta\nu_o = \frac{mv^2}{2h} = \frac{2h\nu_o^2}{mc^2}\left(\frac{1-\cos\theta}{2}\right) - \frac{2h\nu_o\Delta\nu_o}{mc^2}\left(\frac{1-\cos\theta}{2}\right) \tag{114}$$

$$\nu_o - \nu_\theta = \frac{2h\nu_\theta\nu_o}{mc^2}\sin^2\theta/2\,. \tag{115}$$

If we call $\frac{h\nu_o}{mc^2} = \alpha$ this becomes

$$\nu_o - \nu_\theta = 2\alpha\nu_\theta\sin^2\theta/2. \tag{116}$$

In terms of wave-length this becomes

$$\lambda_\theta - \lambda_o = \frac{2h}{mc}\sin^2\theta/2. \tag{117}$$

Substituting the numerical values of h, m, and c, equation (117) becomes

$$\lambda_\theta - \lambda_o = .0484\sin^2\theta/2$$

so that at $\theta = 90°$, $\lambda_\theta - \lambda_o = .024$ Å.

Experimentally the value for $\lambda_\theta - \lambda_o$ at 90° is found by Compton [55] to be .022 Å. and by Ross [56] to be .025 Å.

Since equation (117) has no arbitrary constants by which the value of $\lambda_\theta - \lambda_o$ may be artificially made to coincide with the experimental values, the agreement between theory and experiment is very striking. But it is no more striking than the agreement between the experimental values for intensity of a long

* The relativity correction for the mass of the electron is omitted, for it is evident that it would not appear in equation (114).

wave-length emitted by a very hot "black body" and the theoretical values as calculated from the properties of hydrogen by Rayleigh's formula. Yet Rayleigh's formula does not apply to the whole range of wave-lengths and temperatures and has been supplanted by Planck's formula. Whether or not the success of equation (117) is more than a coincidence must be decided, not merely in terms of the values calculated from this equation alone, but in terms of the success of still other scattering equations derived from it. For this reason we must consider the effect of the angle of scattering upon the intensity of the scattered radiation, and the effect of wave-length on the scattering coefficient. In this way we may hope to gain cumulative evidence for the theory on which equation (117) rests.

Now equation (116) is identical with what we would have had if we had assumed that a spreading wave had hit the electron squarely and had, because of the "momentum" of the wave, given the electron a component of velocity along the direction of wave propagation in addition to the velocity along the electric vector. This would have produced a Doppler effect in the scattered rays such that

$$\nu_o - \nu_\theta = \frac{2\beta}{1-\beta}\nu_\theta \sin^2 \theta/2 \tag{118}$$

where β is the ratio of the velocity of the electron in the direction of the propagation of the wave to the velocity of light.

A comparison of equations (116) and (118) shows that as far as the wave-lengths of the scattered radiation are concerned, we may replace the recoiling electron by one moving in the direction of the propagation of the wave with a velocity,

$$\beta c = \frac{\alpha c}{1+\alpha}$$

If, therefore, we arbitrarily assume that when an electron is hit by a spreading wave, it acquires a velocity

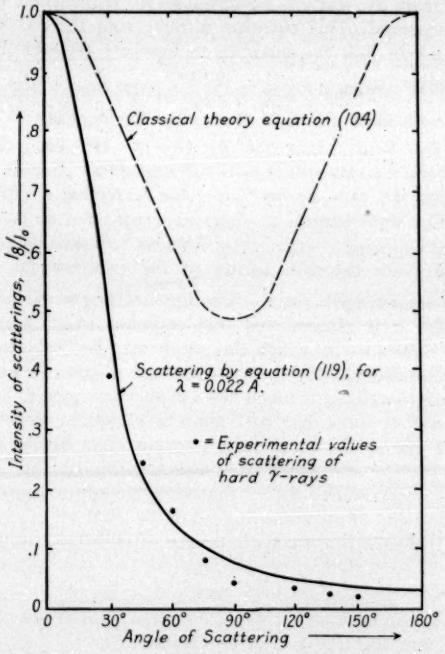
$$\beta c = \frac{h\nu_o c}{mc^2 + h\nu_o}$$

in the direction of propagation of the wave, then the spreading-wave theory leads to the experimental facts as well as does the "dart" theory.

Just as we found before that the great advantage of the Bohr theory over the static-atom theory lay in the fact that it provided a

rational derivation of Thomson's "constant," so now we find that, as far as scattering is concerned, the advantage of the "dart" picture over the spreading-wave picture is in its ability to give a

FIG. 8.



Scattering of gamma rays. Data from Compton. Dotted line from equation (104). Full line from equation (119).

rational derivation of the velocity of the scattering electron in terms of universal constants.

Assuming that the two electrons, one recoiling from a "dart," the other forced ahead by a spreading wave, are equivalent not only in their effect on the wave-length, but also in the variation in the intensity of scattered radiation with the angle of scattering, Compton finds that

(119) $$I_\theta = I\,\frac{nZe^4}{2R^2m^2c^4}\left(\frac{1+\cos^2\theta+2\alpha\,(1+\alpha)\,(1-\cos\theta)^2}{[1+\alpha\,(1+\cos\theta)]^3}\right)$$

where I is the intensity of the primary beam, Z is the atomic number, n the number of atoms per cubic centimetre, and R the distance from the scattering electron to the point where the intensity is measured. This equation differs from the classical equation (104) in that the quantity in brackets replaces the factor $(1+\cos^2\theta)$. Since $\alpha = \frac{h}{mc\lambda_0} = \frac{.24\times10^{-9}}{\lambda_0}$, equation (119) fits the experimental facts for light waves by degenerating to equation (104). For long X-rays like M_{o_k} ($\lambda = .7\times10^{-8}$ cm.) the difference between the two equations is still negligible. For short wave-lengths like W_k ($\lambda = .2\times10^{-8}$ cm.) the correction is quite appreciable. For wave-lengths as short as γ-rays ($\lambda = .02\times10^{-8}$ cm.) the correction is quite large. Fig. 8 shows how well the intensities calculated from equation (119) fit the experimental data for γ-rays of wave-length .022 Å. The agreement is remarkable, especially when it is remembered that equation (119) contains no arbitrary constants by which the graph may be "adjusted."

Since, as has already been pointed out, a dart of radiation of limited cross-section acts much like a material corpuscle, Jauncey [57] has been able to show that, with appropriate assumptions as to the nature of the collision, a purely corpuscular theory will give equation (119). Of course this does not remove the objections to a purely corpuscular theory that have already been mentioned.

On the basis of the spreading-wave theory the energy removed from the primary beam per centimetre of path by scattering is [58]

(120) $$\sigma_0 = \frac{8\pi nZe^4}{3m^2c^4}.$$

It is a consequence of the equations upon which (119) is founded, that the true scattering coefficient is

(121) $$\sigma = \frac{\sigma_0}{1+2\alpha} = \sigma_0\,\frac{1}{1+\frac{2h}{mc\lambda_0}}.$$

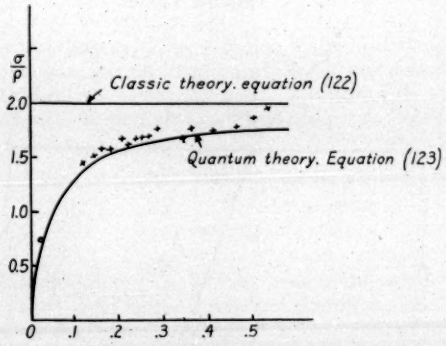
Since the scattering coefficient is usually expressed in terms of the energy removed from the primary beam per gram of the scattering medium per centimetre of path, equations (120) and (121) become

(122) $$\frac{\sigma_0}{\rho} = \frac{8\pi nZe^4}{3m^2c^4\rho}$$

$$\frac{\sigma}{\rho} = \frac{\sigma_0}{\rho} \frac{1}{1 + \frac{2h}{mc\lambda_0}}. \tag{123}$$

Fig. 9 shows how closely the experimental results of Hewlett [59] fit the curve of equation (128). Here again, the agreement is especially striking because of the absence of any adjustable constants in the equation. Fig. 9 shows the results up to a wave-length of .5 Å. We have already seen from Fig. 5 that for wave-lengths

FIG. 9.



Apparent absorption of X-rays due to scattering in carbon. Upper curve from equation (122). Lower curve from equation (123). Data from Hewlett.

longer than this an "excess" scattering occurs due to the electrons in an atom acting as a group. Hewlett's results confirm this, too, for at wave-lengths greater than .5 Å. his data show that more energy is lost by the primary beam than can be accounted for by equation (123).

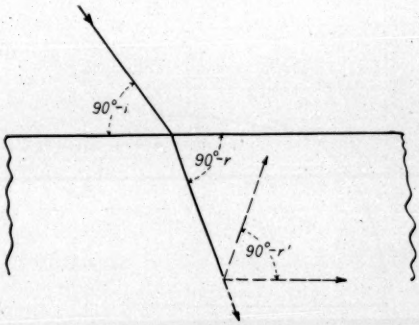
This is not inconsistent with the "dart" theory, for we have assumed that only in the case of very short wave-lengths is the cross-sectional area of a dart comparable with that of an electron. For longer wave-lengths the cross-section is supposed to be greater. If now we assume that for X-ray wave-lengths greater than .5 Å. the area of the dart is comparable to that of an atom, then the excess scattering due to the coöperation of several electrons should be noticeable and equation (105) or (106) should apply.

Refraction.—It is shown in the most elementary text-books that a wave theory of radiation leads to the ordinary law of refraction

$$\frac{\sin i}{\sin r} = \frac{c_i}{c_r} \tag{124}$$

where i and r are the angles of incidence and reflection, respectively, and c_i and c_r are the velocities of propagation in the two media. The method of derivation of this formula applies equally well to the classical spreading-wave theory and to a "dart"-wave

FIG. 10.



theory as long as the area of cross-section of the dart is great enough to include several coöperating electrons at once as the dart passes through the refractive medium.

With the aid of the quantum theory, Duane [60] has shown that equation (124) holds also for darts of very small cross-section or even for a strictly corpuscular radiation. Let the dart or corpuscle strike the refracting surface (Fig. 10) at an angle $90° - i$ with a velocity c_i. Then its momentum before impact is $\frac{h\nu}{c_i}$. The component of this momentum parallel to the refracting surface is $\frac{h\nu}{c_i} \sin i$. Similarly the component in the same direction after refraction is $\frac{h\nu}{c_r} \sin r$. Then

$$\frac{h\nu}{c_i} \sin i - \frac{h\nu}{c_r} \sin r = M \tag{125}$$

where M is the momentum imparted to the material substance (electrons) at the surface.

Since h has the dimensions of a momentum times a length,

$$M = n\frac{h}{l} \tag{126}$$

where l is a length in the direction in which the momentum is considered, and where n is a pure number, usually small, but not necessarily an integer. Combining equations (125) and (126), we have

$$\frac{h\nu}{c_i}\sin i - \frac{h\nu}{c_r}\sin r = \frac{nh}{l}. \tag{127}$$

Now l is a distance in the direction of the refracting surface. The only distance in this direction which has any meaning is the length of the refracting surface itself. l is ordinarily so large in comparison with nh that nh/l approximates zero. Therefore

$$\frac{h\nu}{c_i}\sin i = \frac{h\nu}{c_r}\sin r \tag{128}$$

or

$$\frac{\sin i}{\sin r} = \frac{c_i}{c_r}$$

When l is comparable to the spacings used in diffraction gratings, equation (128) is no longer a sufficient approximation.

Just after the "dart" (or corpuscle) has entered the refracting medium, it may be deflected, so that now its path is $90° - r'$ to the surface. Corresponding to equation (127), we have

$$\frac{h\nu}{c_r}(\sin r - \sin r') = \frac{n_1 h}{l} \tag{129}$$

$$\frac{h\nu}{c_r}(\cos r - \cos r') = \frac{n_2 h}{t} \tag{130}$$

where t is the only distance perpendicular to the surface which has any meaning, namely, the thickness of the refracting substance. If l in equation (129) is large in comparison with h, then $\sin r = \sin r'$. This means that either $r' = r$ or $r' = 180° - r$. Every dart for which $r' = r$ must pass on through the body of the refracting medium without change in direction. If $r' = 180° - r$ and if n_2 is zero, then equation (130) shows that no momentum is imparted to the refracting substance in the direction of t and the "dart" is reflected out of the substance according to the ordinary laws of

elastic impact so that the angle of incidence equals the angle of reflection.

If $r' = 180° - r$ and if n_2 is not zero, then equation (130) becomes

$$n_2\lambda' = 2t\cos r \tag{131}$$

where λ' is the wave-length inside the refracting medium. If n_2 is an integer, this is the ordinary equation for the interference of light from a thin plate. If n_2 is not an integer, momentum may be imparted to the refracting substance, so that the energy may finally appear as fluorescent radiation, scattered radiation or heat, thus reducing the effective value of n_2 to the next smaller integer.

Equation (131) is the same as the equation in the classical theory for "destructive interference." If the use of half-quanta be permitted, the corresponding equation for "constructive interference" (reflection) may be obtained. The idea of a half-quantum seems to be contradictory to the whole original quantum hypothesis, but this is by no means the only instance where its use is necessary to make the theory fit the experimental facts.

Interference.—Since the beginnings of the spreading-wave theory it has been regarded as one of the strong points of the theory that it explained interference. When two portions of the same wave were led along two different paths and were later re-united, then if they met crest-to-crest ("constructive interference") the original wave was restored; if they met crest-to-trough ("destructive interference") the wave was extinguished. This explanation seems to be consistent with all the facts known at present. It may be found in the textbooks in varying mathematical form to suit nearly every conceivable sort of path for the two portions of the original light wave. The original wave may be split by half-silvered mirrors as in the interferometer or by two mirrors placed so as to face each other (Fresnel mirrors), or by two prisms placed back to back (Fresnel bi-prism), or by a diffraction grating. In every case the experimental result may be accurately predicted by the spreading-wave theory. Even the "interference pattern" of X-rays from a three-dimensional grating (crystal) may be calculated by the spreading-wave theory to give the same result as is obtained by experiment.

It has until recently been one of the greatest objections to any form of corpuscular theory that it seemed impossible for cor-

puscles to form an interference pattern. That objection has not yet been completely overcome, but enough progress has been made to show that interference of the "dart" form of corpuscles is not altogether impossible. For instance, if half of each dart is transmitted through the mirror of an interferometer, and the other half is reflected, we should have all the conditions necessary for interference. This would mean that radiant energy may travel along the split path in half-quanta. Such an idea is not inconsistent with the original idea of quanta, for the quantum was considered to owe its existence to the conditions existing inside the radiating atom, and not to the nature of the method of propagation of radiation. As stated above, several phenomena seem to require half-quanta if they are to be explained on the basis of the quantum theory.

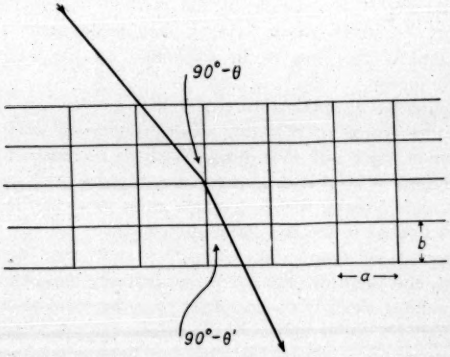
An alternative explanation for "half-quanta" would be to assume that groups of radiating electrons give off darts in pairs which are in phase with each other. We have no reason to suppose that electrons in neighboring atoms are in phase with each other, but it is quite possible that in a single atom some of the electrons in a given shell (to use the static-atom picture) vibrate in phase. Measurements of atomic dimensions [22, 43] show that the diameter of atoms, and therefore the maximum distance between electrons in the outmost shell, is of the order of a ten-thousandth of the wave-length of visible light. If, therefore, the vibrations of certain electrons are in phase, the waves of light will be sensibly in phase. If these electrons send out "darts" in such directions that in every case two travel side by side, the pairs might represent the quantum and a single "dart" would represent a "half-quantum."

So far, there seems to be no explanation of the action of Fresnel mirrors or a Fresnel bi-prism in terms of a corpuscular theory. We might, of course, make the improbable assumption that two darts coming from the same source in slightly divergent paths are in phase with each other. In such a case the mirrors or bi-prism should produce diffraction only when the distance to the source of light falls within definite limits. As far as is known to the writer, this has never been tried experimentally. We could hardly assume that the cross-section of a dart increased as the distance from the source is increased, for if we assume a wide enough solid angle to "explain" laboratory experiments with

mirrors or a bi-prism, we would be unable to show why a photo-electric cell can be operated with light from a star.

Using the same assumptions as in the discussion of refraction, Duane [60] has shown that a corpuscular radiation (and therefore "darts") can be diffracted by a grating according to the same laws that are found for a spreading wave. If the diffraction grating is an ordinary ruled grating such as is used in experiments with visible light, then l of equation (127) is the distance, d, between the lines of the grating. Since we are not dealing with refraction,

FIG. 11.



the velocity of the emergent beam is the same as that of the incident beam. Equation (127) therefore becomes

$$\frac{h\nu}{c}\sin i - \frac{h\nu}{c}\sin r = \frac{nh}{d}$$

$$\frac{nc}{\nu} = n\lambda = d(\sin i - \sin r) \tag{132}$$

which is the ordinary equation for the action of a plane ruled grating if n is an integer. If n is not an integer, the "dart" may give up energy to the grating to appear later as scattered radiation or heat, thus reducing the effective value of n to the next smaller integer.

The case of X-rays diffracted by a crystal is equally simple. Let Fig. 11 be the cross-section of a crystal whose axes are all at right angles to each other and whose atoms lie at the intersections

of the lines. Let a and b be the distance between parallel planes in the x and y directions, respectively, and let an X-ray "dart" (Fig. 11) pass into the crystal at an angle $90° - \theta$ to the surface of the crystal. At some point in its path where it encounters an electron the "dart" may be deflected, so that it emerges from the crystal at an angle $90° - \theta'$ to the surface. Then the momentum transferred to the plane of atoms in the crystal parallel to the X-axis is

$$\frac{h\nu}{c}\left(\sin\theta - \sin\theta'\right) = \frac{n_1 h}{a}. \tag{133}$$

The momentum transferred to the plane of atoms parallel to the Y-axis is

$$\frac{h\nu}{c}\left(\cos\theta - \cos\theta'\right) = \frac{n_2 h}{b}. \tag{134}$$

As in the case of the diffraction of visible light, n_1 and n_2 must be integers if equations (133) and (134) are to be applied to diffraction. If they are not integers, the crystal must take energy in some way from the "dart," either by emitting characteristic rays or by scattering, etc., until the momentum of the "dart" is reduced sufficiently so that n_1 and n_2 are both integers.

If n_1 and n_2 are both zero, the "dart" passes through the crystal without change in direction until it strikes another electron. If n_1 is zero and n_2 is some integer other than zero, $\sin\theta = \sin\theta'$, but $\cos\theta$ does not equal $\cos\theta'$. Therefore, $\theta' = -\theta$. Substituting in equation (134) we have

$$\frac{2h\nu}{c}\cos\theta = n_2\frac{h}{b}$$

or

$$n_2\lambda = 2b\cos\theta. \tag{135}$$

This is the ordinary Bragg law of X-ray diffraction from the X-planes. Similarly if n_1 is some integer other than zero and if n_2 is zero,

$$n_1\lambda = 2a\sin\theta \tag{136}$$

which is the Bragg law for diffraction from the Y-planes.

If n_1 and n_2 are both integers other than zero, equations (133) and (134) show diffraction due to other planes of atoms in the crystal.

The changes which must be made in the above discussion in the case of crystals whose axes are not at right angles to each

other, are obvious. These changes complicate the equations but do not change the argument nor the conclusions.

It is, therefore, evident that, by making the proper assumptions, the effect of diffraction from a grating may be "explained" on the basis of a "dart" (or corpuscular) theory of radiation about as well as on the basis of spreading waves.

SUMMARY.

A large mass of physical and chemical data seems to point toward a static-atom picture of atomic structure such as is given by the Lewis-Langmuir atom. Data from radiation seem to point toward the Bohr-Sommerfeld type of picture. By making appropriate assumptions as to the law of electrostatic force between unlike charges and by assuming the quantum theory the two pictures give identical results. There is reason to suppose that they are really two different viewpoints of the same picture. In this way we arrive at rather definite ideas as to the source of the radiation.

Most experiments which deal with the material substance which finally receives the radiation are consistent with either a corpuscular or a spreading electromagnetic wave picture of radiant energy. The photo-electric effect and the photographic effect seem to be quite inconsistent with a spreading-wave theory, but are consistent with any form of corpuscular theory.

Experiments which deal with the scattering effect of material substances placed in the path of radiation are consistent with a spreading-wave theory only if we make the proper assumptions as to the velocity of the scattering electron. They are all consistent with the electromagnetic dart theory and with any form of corpuscular theory. Data on absorption and transmission, together with the absence of any effect on radiation by a pure electrostatic or magnetic field, make only one form of corpuscular theory tenable, namely, Silberstein's "dart theory." These darts are pictured as trains of electromagnetic waves of tiny cross-section so that each "dart" is an entity in itself.

The "dart" theory does not deny the existence of spreading electromagnetic waves such as are found in radio work, but merely assumes that the mechanism of radiation from the electrons in atoms and molecules is such as to produce tiny beams which, by

reason of some focussing action in the atom, are propagated with little or no change in cross-section.

Such a theory is consistent with all known experimental facts except one, namely, diffraction from Fresnel mirrors, and what amounts to the same thing, diffraction from a bi-prism.

In view of the recent rapid progress of such a theory due to the work of Duane and A. H. Compton, it is not too much to hope that the single remaining objection may be soon overcome.

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The Boiling Point of Bromine. A. BOUZAT and G. LELUAN. (*Comptes Rendus*, Feb. 11, 1924.)—In nine carefully conducted experiments the lowest boiling point found was 58.77° and the highest 58.84° C. The final value given is 58.80° C. This is .05° higher than the number resulting from the work of Ramsay and Young.

G. F. S.

THE SCATTERING OF X-RAYS.*

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ABSTRACT.

Present Status of the Problem.—The scattering of X-rays is one of the outstanding problems of electromagnetic radiation which has not been solved satisfactorily. All theories (based on classical electrodynamics) presented thus far do not explain either the diminution in the scattering coefficient, or the observed asymmetry in the scattering, or both. Among such theories we may mention:

J. J. Thomson's Theory.—Assuming that the scattering is done by a point electron, and making use of certain additional hypotheses, Thomson showed that the scattering coefficient of any substance is given by

$$\sigma = \frac{8\pi N p e^4}{3m^2c^4},$$

and that the intensity of the scattered radiation is given by

$$I_\theta = I\frac{e^4(1+\cos^2\theta)}{2r^2m^2c^4},$$

where N is the number of atoms per c.c., p the number of electrons per atom, e the electronic charge, m the electronic mass, c the velocity of light, I_θ the intensity of the scattered radiation at an angle θ between the incident beam and the radius vector joining the centre of the electron and the point P distant r from the electron, and I is the intensity of the incident beam. This theory explains neither the asymmetry nor the decrease in the coefficient of scattering.

Schott's Theory.—Among other things, the assumption is here made that the atom consists of coaxal rings of electrons. The electrons in each ring are spaced at equal intervals and revolve with a uniform angular velocity, which, however, may be different for different rings. This theory fails to explain the observed diminution in the scattering coefficient.

Debye's Theory.—In its essentials, Debye's theory has the same merits and demerits as that of Schott. Debye assumes that all the electrons in an atom are arranged in a single ring, and that they are spaced at equal intervals. This theory (and also Schott's theory) explains the asymmetry and the "excess scattering," but is altogether unable to explain the diminution in the scattering coefficient.

Modification of the Classical Theory.—The present paper presents a discussion of the possibility of modifying the classical theory (that of J. J. Thomson) so as to account for the decrease in the scattering coefficient as well as the dissymmetry. By assuming that the electron is made up of a number of parts—for simplicity, of two parts—it has been found possible

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to account for the diminution in the scattering coefficient without, at the same time, explaining the observed asymmetry. To accomplish both objects is what was aimed at in the *combination of the present work with that of Debye*. In this research the goal has not been perfection between predicted and observed results, but rather to discuss some possible modifications of the classical theory and their consequences.

INTRODUCTION AND BRIEF REVIEW OF SOME PREVIOUS WORK.

1. One of the outstanding problems in electromagnetic radiation which thus far has not received a satisfactory solution is the one offered by the phenomenon of the so-called "scattered X-rays." Many attempts have been made, on the basis of classical electrodynamics, to derive an expression giving the correct amount of energy scattered by various elements under various conditions (*e.g.*, for different wave-lengths). However, it cannot be said that any of these attempts has met with success.

2. The first attempt to give a theoretical explanation of the scattering of X-rays was that of Sir J. J. Thomson. On assuming that

(1) Classical electromagnetic theory is applicable to the problem;
(2) Each electron scatters independently;
(3) There are no other forces acting on the electrons which are comparable in magnitude with the forces due to the incident beam;
(4) The dimensions of the electron are negligible compared with the wave-length of the incident radiation,

Professor Thomson [1] showed that the mass scattering coefficient of any substance is given by the expression

$$\sigma = \frac{8\pi N p e^4}{3m^2c^4}, \tag{1}$$

where N is the number of atoms per c.c., p the number of electrons per atom, e the electronic charge, m the mass of an electron, and c the velocity of light. According to Thomson's theory, the intensity of the beam scattered by an electron at an angle θ with the incident beam is given by

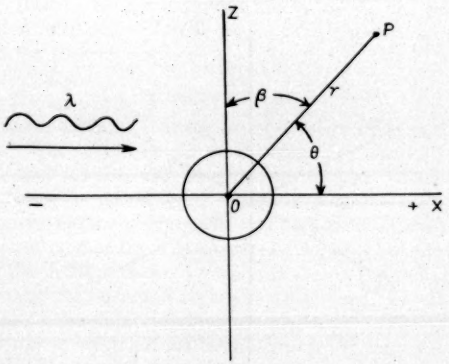
$$I_\theta = I\frac{e^4(1+\cos^2\theta)}{2r^2m^2c^4}, \tag{2}$$

where I is the intensity of the incident beam, and r the distance between the centre of the electron and the point at which we are calculating the intensity of the scattered beam (Fig. 1).

3. From formulas (1) and (2) the conclusions are drawn that (*a*) the coefficient of scattering should be independent of the wave-length; (*b*) the mass scattering coefficient should not in any case be less than .20 if the number of electrons in atoms other than those of hydrogen is equal to the atomic number, and (*c*) the distribution of the intensity of the scattered beam should be symmetrical with respect to the scattering plate (radiator).

4. The experiments of Barkla and Dunlop [2] show that for a considerable range in wave-lengths of X-rays the mass scatter-

FIG. 1.

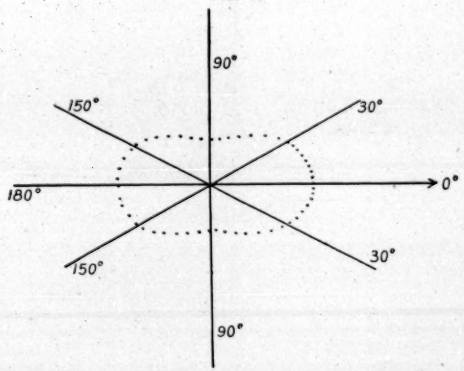


ing coefficients of elements of low atomic weight are correctly given by (1) if the number of electrons is taken to be equal to the atomic number. This assumption has been shown [2] to be valid for the lighter elements when X-rays of ordinary hardness are used. But, for elements of high atomic weight and soft X-rays, the total scattering is much greater than would be expected from Thomson's formula. On the other hand, Barkla and White [3] have shown that for wave-lengths less than 2×10^{-9} cm. the total mass absorption coefficient of a light element is less than the theoretical value of the mass scattering coefficient alone. Moreover, Ishino [4] has shown that in the case of radiation of high frequency the value of the mass scattering coefficients for Al, Fe, and Pb are only one-fourth of the values calculated on the basis of Thomson's theory.

5. The fact that, when soft X-rays are used, elements of high atomic weight scatter to a greater extent than would be predicted by the classical theory has been accounted for by assuming [5] that the electrons do not scatter independently when the wave-length of the incident radiation is comparable with the dimensions of the atom. But classical electrodynamics has not been able to account for the diminution in the scattering when hard X-rays are used.

6. It has already been noticed that according to Thomson's theory the intensity of the scattered radiation should be sym-

FIG. 2.



Theoretical distribution of the intensity of scattered X-rays according to Thomson's theory.

metrically distributed with respect to the radiator, as shown in Fig. 2. For those elements of low atomic weight whose mass scattering coefficients are given correctly by (1), Barkla [6] has shown that for a certain range in wave-lengths this prediction is fulfilled. But, for relatively soft X-rays and hard X-rays this prediction has been shown [7] not to be valid.

7. When heavy elements and incident radiation of long wave-lengths are used, the dissymmetry is accompanied by an *increase* in the total energy scattered. This "excess scattering" is usually explained by assuming that in heavy elements the electrons are "closely packed," so that the rays scattered by the individual electrons are almost in the same phase.

8. However, this explanation cannot be applied to the asymmetrical scattering of hard γ-rays.[8] For the asymmetry in this case is accompanied by a *diminution* in the total energy scattered and not an *increase* as we should expect. As Compton[8] points out, one is thus led to the conclusion that the asymmetrical scattering of short waves is due to some property of the electron itself.

9. The general problem of the scattering of X-rays by groups of electrons has been studied by Debye,[9] Schott,[10] Glocker and Kaupp,[11] and A. H. Compton.[12] In their essential features the theories of these investigators do not differ greatly. We shall therefore discuss briefly only the theory of G. A. Schott and rather more fully that of Debye, as we shall make use of the latter theory in the present work.

SCHOTT'S THEORY.

10. Schott assumes that the atom consists of coaxal rings of electrons. The electrons in each ring are spaced at equal intervals, and revolve with a uniform angular velocity which, however, may be different for different rings. In this theory, as well as in those of Thomson and Debye, it is assumed that classical electrodynamics is applicable. The effect of the magnetic forces in the incident beam, the reaction due to the radiation from the electrons, the variation of mass with speed, and the mutual interaction of the different rings are neglected. The assumption that the rings of electrons scatter independently makes it unnecessary to take into account the multiple scattering which undoubtedly occurs to some extent and which would have the effect of diffusing the radiation and so decrease the asymmetry.

11. The final results arrived at are that (1) in the case of hydrogen the predictions concerning the scattering coefficient and asymmetry are the same as those of the classical theory; (2) for atoms containing rings of more than one electron there should be fore and aft asymmetry, and the scattering coefficient should increase with increasing wave-length up to a value not much less than p times that given by the classical theory, where p is the number of electrons in the ring; (3) the character of the scattering and asymmetry depends on the number of rings in the atom and the number of electrons in each ring; (4) the minimum value of the scattering coefficient on this theory is that given by the classical theory; and (5) this theory is altogether incapable of

explaining the exceptionally small scattering coefficients observed by Ishino.[4]

DEBYE'S THEORY.

12. Debye's theory, in its essentials, has the same merits and demerits as that of Schott. It also accounts for the dissymmetry and the excess scattering, but is totally unable to account for the small scattering coefficients observed by Ishino, Barkla, and others.

13. Debye assumes that all the electrons in an atom are arranged in a single ring of radius a, and that they are spaced at equal intervals. He further assumes that the only effective forces are those of the incident beam.

14. Let the incident beam, which is assumed to be polarized in the Z-direction, be directed along the X-axis of a right-handed coördinate system. The components of the electric vector as given by Debye[9] are:

$$E_x = -\alpha\gamma \frac{e^2}{mc^2} E_0 \frac{\varepsilon^{i(\omega t - kR)}}{R} \cdot \varepsilon^{ik\eta_n},$$

$$E_y = -\beta\gamma \frac{e^2}{mc^2} E_0 \frac{\varepsilon^{i(\omega t - kR)}}{R} \cdot \varepsilon^{ik\eta_n}, \tag{3}$$

$$E_z = (1-\gamma^2) \frac{e^2}{mc^2} E_0 \frac{\varepsilon^{i(\omega t - kR)}}{R} \cdot \varepsilon^{ik\eta_n}$$

where

$$\eta_n \equiv [(\alpha - 1)x_n + \beta y_n + \gamma z_n]; \quad k \equiv \omega/c,$$

R is the distance from the centre of the ring to the point at which the intensity is being calculated, α, β, γ are the direction-cosines of the vector joining this point and the origin, e the electronic charge, ε the Naperian base, and x_n, y_n, z_n the coördinates of the nth electron in its position preceding the disturbance by the incident beam. For the whole field and at great distances we will have[9]

$$E_x = -\alpha\gamma \frac{e^2}{mc^2} E_0 \frac{\varepsilon^{i(\omega t - kR)}}{R} \cdot \sum_1^p \varepsilon^{ik\eta_n},$$

$$E_y = -\beta\gamma \frac{e^2}{mc^2} E_0 \frac{\varepsilon^{i(\omega t - kR)}}{R} \cdot \sum_1^p \varepsilon^{ik\eta_n}, \tag{4}$$

$$E_z = (1-\gamma^2) \frac{e^2}{mc^2} E_0 \frac{\varepsilon^{i(\omega t - kR)}}{R} \cdot \sum_1^p \varepsilon^{ik\eta_n}.$$

The energy is proportional to E^2 where

$$E^2 = E_x^2 + E_y^2 + E_z^2. \tag{5}$$

Debye[9] has shown that, if subscripts n and m refer to the nth

and mth electrons, the ratio of the intensity of the radiation scattered by one atom at a large distance R, to that of the energy contained per square centimetre in the incident beam is given by

$$v = \frac{e^4(1+\alpha^2)}{2R^2m^2c^4} \sum\sum \epsilon^{ik[(\alpha-\alpha)(x_n-x_m)+\beta(y_n-y_m)+\gamma(z_n-z_m)]}. \quad (6)$$

15. In order to find the scattering by many molecules with electron rings we note first that for an isomorphous body, all orientations of the atoms (molecules) are equally likely. Thus, if we consider a portion of the radiator, say one c.c., in which there are N atoms, the intensity of the scattered radiation in a direction whose direction-cosines are α, β, γ, may be found by first finding the mean value of v for one atom for all possible orientations and then multiplying the result by N. The final result obtained by Debye is that the ratio of the intensity of the energy scattered by N atoms to that of the incident beam is given by

$$V = \frac{Npe^4(1+\alpha^2)}{2R^2m^2c^4} \sum_{n=0}^{p-1} \frac{\sin\left[4ka\sin\frac{n\pi}{p}\sin\frac{\theta}{2}\right]}{\left[4ka\sin\frac{n\pi}{p}\sin\frac{\theta}{2}\right]}, \quad (7)$$

where θ is the angle between the incident beam and the direction at which V is calculated, N the number of atoms per c.c., and p the number of electrons per atom.

16. Let λ be the wave-length of the incident radiation. Then, from the definition of k, we will have

$$ka = \frac{2\pi a}{\lambda}. \quad (8)$$

The following special cases are of interest:

Case 1: If λ is large in comparison with the radius of the ring of electrons (which in the case of hydrogen is about $.72 \times 10^{-8}$ cm.) so that $ka<<1$, then each term under the summation sign in (7) approaches 1, and

$$V = \frac{(1+\cos^2\theta)\,e^4Np^2}{2R^2m^2c^4}. \quad (9)$$

Case 2: On the other hand, if $ka>>1$, *i.e.*, if $\lambda<<a$ and θ is sufficiently large,

$$V = \frac{(1+\cos^2\theta)\,e^4Np}{2R^2m^2c^4} \quad (10)$$

17. The coefficient of scattering, σ, for the two special cases may be found by integration. The results are:

Case 1: $\lambda >> a$.

$$\sigma = \frac{8\pi e^4 N p^2}{3m^2c^4} \tag{11}$$

Case 2: $\lambda << a$.

$$\sigma = \frac{8\pi e^4 N p}{3m^2c^4} \tag{12}$$

In the general case σ will be given by

$$\sigma = \int VR^2 d\Omega , \tag{13}$$

where $d\Omega$ is an element of solid angle.

18. The minimum amount of scattering according to Debye's theory is far in excess of the observed values. However, as has been mentioned, the theory furnishes a plausible explanation of the fore and aft asymmetry and the accompanying excess radiation. The dissymmetry will depend on the number of electrons in the ring. The function

$$\psi_p(j) = \frac{1}{p} \sum_{o}^{p-1} \frac{\sin\left[j \sin \frac{n\pi}{p}\right]}{\left[j \sin \frac{n\pi}{p}\right]},$$

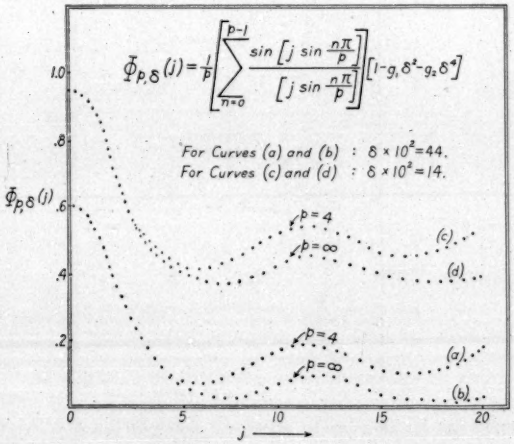
where $j = 4ka \sin \frac{\theta}{2}$, when plotted against j would have the same form as the curves given in Fig. 3, except that each ordinate would be magnified slightly and by an equal amount. Fig. 4 shows the theoretical distribution of intensity according to Debye's and Thomson's theories. In Fig. 5 Crowther's [13] experimentally determined curve for the distribution of intensity round an Al radiator is compared with the theoretical distribution according to Debye's theory when hydrogen is used as the scattering substance.

19. We have seen that experiments on the scattering of short waves lead us to the conclusion that the explanation of the small scattering coefficients is to be found in some property of the electron itself. An interesting attempt to do this is that of A. H. Compton.[8] Compton assumes that the radius of the electron is of the same order of magnitude as the wave-length of short γ-rays, and that the incident electromagnetic wave is capable of moving the different parts of the electron relatively to each other. He further postulates the validity of assumptions (1) to (3) given in paragraph 2. The size of the electron being comparable with the wave-length used, the wave scattered by different parts of

the electron will not all be in exactly the same phase. The introduction of the phenomenon of interference is found to lead to a decrease in the scattering coefficient and also to a fore and aft asymmetry in the distribution of the intensity of the scattered radiation.

20. Among the objections one might raise against the assumption of a large electron are that (1) recent experiments by Richtmyer [14] have given values of the mass absorption coefficient

FIG. 3.

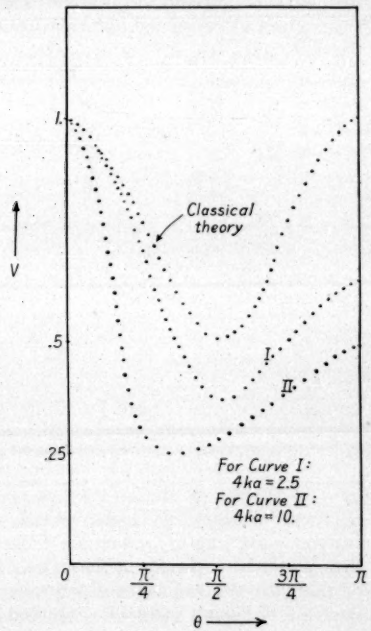


for Al (for different wave-lengths) which are from 7 to 90 per cent. higher than would be expected on the basis of Compton's theory; (2) if we take the radius of Compton's electron as 2×10^{-10} cm. and use Millikan's value of e, we find the mass of the electron to be only 1/1000 that obtained from the values of e and e/m; and (3) in assuming that each element of the electron has a definite mass independent of the rest of the electron and that there is relative motion among the parts of the electron, Professor Compton cannot call upon classical electrodynamics to guide him in finding the motion of his electron. For, classical electrodynamics gives us no information as to how an element of an electron moves. As regards the motion of the electron as

a whole, the usual assumption is that of Lorentz, namely, that the electron moves in such a way that the external force acting on it is equal and opposite to that which the electron exerts on itself.

21. In the present reign of the quantum theory one would naturally turn to it with the object of deriving whatever benedic-

FIG. 4.



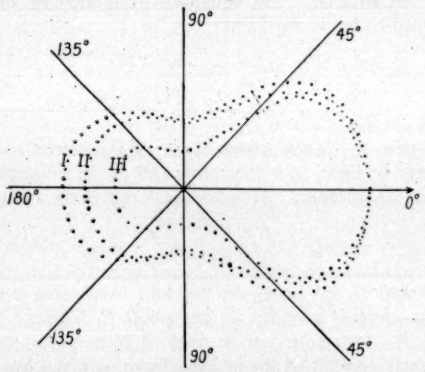
Intensity of scattered X-rays as a function of θ. (For hydrogen.)

tion it has so far as the scattering of X-rays is concerned. Without invoking hypotheses founded on the quantum theory, however, it would be of interest to see whether the classical theory could be modified in such a way as to be able to explain the outstanding facts of the scattering of X-rays, in particular, the observed diminution in the scattering coefficient. The object of

the present work is to present a discussion of the possibility of doing this.

22. We shall assume that the electron consists of a large number of parts—for analytical simplicity, of two parts—having equal charges but different masses. In addition, as regards the

FIG. 5.



Distribution of the intensity of scattered X-rays.
I: Theoretical curve for hydrogen according to Debye's theory ($4ka=2.5$).
II: Theoretical curve for hydrogen according to the modified Debye's theory ($4ka=2.5$).
III: Crowther's experimental curve for Al.

electron as a whole, we make the assumptions stated above when reviewing J. J. Thomson's theory.

23. We recall that the equation of motion of an electron is usually written as

$$(\text{mass}) \cdot (\text{acceleration}) = \text{external force}.$$

We shall modify this equation for each part of the electron by adding to the left-hand side a term depending on the relative displacements, and another term depending on the relative velocities of these parts. The introduction of the term depending on the relative velocities corresponds to introducing, in ordinary dynamics, a frictional term. Our usual ideas of friction, however, are altogether inapplicable when we consider parts of an electron. Nevertheless, this term plays the same rôle as the dissipativity function in dynamics and gives rise to some difficulties in connection with the energy equation. A brief discussion of

these difficulties will be given below; for the present we shall give the workable features of the theory in some detail.

24. Let the incident beam, assumed to be polarized in the Z-direction, be directed along the X-axis of a right-handed system of coördinates (Fig. 1). Let m_1, m_2, ξ_1, ξ_2, $(e/2)$, $(e/2)$ be the masses, displacements, and charges, respectively, of the two parts of the electron. The equations of motion of these parts are assumed to be of the form:

$$m_1\ddot{\xi}_1 + b(\dot{\xi}_1 - \dot{\xi}_2) + a(\xi_1 - \xi_2) = \frac{eE_0}{2}\varepsilon^{i(\omega t - kx)}, \tag{14}$$

$$m_2\ddot{\xi}_2 + b(\dot{\xi}_2 - \dot{\xi}_1) + a(\xi_2 - \xi_1) = \frac{eE_0}{2}\varepsilon^{i(\omega t - kx)}, \tag{15}$$

where $k = 2\pi\nu/c$, ν is the frequency of the incident radiation, c the velocity of light, a, b constants, and the other symbols have their usual significance. To solve these equations, assume

$$\xi_1 = A\,\varepsilon^{i(\omega t - kx)}, \tag{16}$$

$$\xi_2 = B\,\varepsilon^{i(\omega t - kx)}, \tag{17}$$

where A and B are to be determined. According to classical theory the electric intensity, at any point P distant r from the centre of the electron and at an angle β between the vector OP and the electric vector of the incident beam, is given by

$$X_P = \frac{e\sin\beta}{2rc^2}(\ddot{\xi}_1 + \ddot{\xi}_2), \tag{18}$$

The amplitude at the point P is given by the expression

$$J = -\frac{e\sin\beta}{2rc^2}\omega^2(A + B) = g\omega^2(A + B), \tag{19}$$

where, for brevity, we have put

$$g \equiv -e\sin\beta/2rc^2. \tag{20}$$

From equations (14), (15), (16), and (17) we get the following equations:

$$-m_1A\omega^2 + b(iA\omega - iB\omega) + a(A - B) = \frac{eE_0}{2}; \tag{21}$$

$$-m_2B\omega^2 + b(iB\omega - iA\omega) + a(B - A) = \frac{eE_0}{2}; \tag{22}$$

i.e.,

$$(A + B) = \frac{-eE_0}{m_2\omega^2} + (1 - \mu)A, \tag{23}$$

where

$$u \equiv \frac{m_1}{m_2}.$$

Solving equations (21), (22), and (23) for $(A+B)$ we find, after a simple calculation, that

$$J = g\omega^2(A+B)$$

$$= \frac{g\omega^2 eE_0\left\{\left[a(1+\mu^2+6\mu)-(1+\mu)\left(m_1\omega^2+\frac{4a^2}{m_2\omega^2}+\frac{4b^2}{m_2}\right)\right]^2+b^2\omega^2(1-\mu)^4\right\}^{\frac{1}{2}}}{2\left[a^2(1+\mu)^2-2am_1\omega^2(1+\mu)+m_1\omega^4+b^2\omega^2(1+\mu)^2\right]}$$

$$= \frac{-e^2\omega^2E_0\sin\beta}{4\pi c^2}Q, \qquad (24)$$

where

$$Q \equiv \frac{\left\{\left[a(1+\mu^2+6\mu)-(1+\mu)\left(m_1\omega^2+\frac{4a^2}{m_2\omega^2}+\frac{4b^2}{m_2}\right)\right]^2+b^2\omega^2(1-\mu)^4\right\}^{\frac{1}{2}}}{a^2(1+\mu)^2-2am_1\omega^2(1+\mu)+m_1\omega^4+b^2\omega^2(1+\mu)^2}. \qquad (25)$$

25. The intensity of the radiation at the point P will be proportional to J^2; and the ratio of this intensity to that of the incident beam will be given by

$$v = \frac{e^4\omega^4\sin^2\beta}{16r^2c^4}Q^2. \qquad (26)$$

If the incident beam is unpolarized the electric vector will be uniformly distributed in the plane of the wave, so that the mean value of $\sin^2\beta$ should be used in calculating v. This is easily shown to be equal to $(1+\cos^2\theta)/2$, where θ is the angle between the incident beam and the vector joining the centre of the electron and the point at which we are calculating v. Hence, for unpolarized light,

$$v = \frac{(1+\cos^2\theta)\,e^4\omega^4}{32\pi^2c^4}Q^2. \qquad (27)$$

The scattering coefficient per electron, σ_1, is found by integrating the last expression over a sphere of radius r. The result is

$$\sigma_1 = \frac{8\pi e^4}{3c^4}\left(\frac{\omega^4Q^2}{16}\right), \qquad (28)$$

where Q depends only on the constants a, b, and the ratio, μ, of the masses m_1 and m_2.

26. To find the value of Q we proceed as follows: Let there be no external force. The equations of motion of the two parts of the electron will then be:

$$m_1\ddot{\xi}_1 + b(\dot{\xi}_1-\dot{\xi}_2) + a(\xi_1-\xi_2) = 0, \qquad (29)$$

$$m_2\ddot{\xi}_2 + b(\dot{\xi}_2-\dot{\xi}_1) + a(\xi_2-\xi_1) = 0. \qquad (30)$$

Dividing equation (29) by m_1 and equation (30) by m_2, and then subtracting (30) from (29), there results:

$$\frac{d^2}{dt^2}(\xi_1-\xi_2) + b\left(\frac{m_1+m_2}{m_1m_2}\right)\frac{d}{dt}(\xi_1-\xi_2) + a\left(\frac{m_1+m_2}{m_1m_2}\right)(\xi_1-\xi_2) = 0. \qquad (31)$$

27. Let

$$m = m_1 + m_2,$$

and

$$\xi = \xi_1 - \xi_2. \tag{32}$$

Then,

$$m_1 = \frac{\mu m}{1+\mu},$$

$$m_2 = \frac{m}{1+\mu} \tag{33}$$

and

$$m_1 m_2 = \frac{\mu m^2}{(1+\mu)^2}.$$

For convenience, place

$$\Delta = \frac{(1+\mu)^2}{\mu m}. \tag{34}$$

Equation (31) may now be written as

$$\frac{d^2\xi}{dt^2} + b\Delta \frac{d\xi}{dt} + a\Delta\xi = 0. \tag{35}$$

The solution of this equation is

$$\xi = C_1 \varepsilon^{-\frac{b\Delta t}{2} + \frac{t}{2}\sqrt{(b\Delta)^2 - 4a\Delta}} + C_2 \varepsilon^{-\frac{b\Delta t}{2} - \frac{t}{2}\sqrt{(b\Delta)^2 - 4a\Delta}} \tag{36}$$

where C_1 and C_2 are constants. If the motion is to be oscillatory, we must have

$$4a\Delta > \Delta^2 b^2.$$

Let

$$q = \frac{1}{2}\sqrt{4a\Delta - \Delta^2 b^2}.$$

Then the appropriate solution of equation (35) may be written in the form

$$\xi = \varepsilon^{-\frac{b\Delta t}{2}} [G_1 \cos qt + G_2 \sin qt], \tag{37}$$

where G_1 and G_2 are constants. By a well-known procedure equation (37) may be written in the more compact form:

$$\xi = G\varepsilon^{-\frac{b\Delta t}{2}} \cos(qt - \varphi), \tag{38}$$

where G and φ are constants. The ratio of two successive amplitudes is $\varepsilon^{\frac{b\Delta t}{2}}$. Introducing the logarithmic decrement, κ, and the

period of free vibration of the oscillator, τ, we find that the constants a and b are given by

$$a = \frac{\pi^2 + \kappa^2}{\Delta\tau^2}, \tag{39}$$

$$b = \frac{2\kappa}{\Delta\tau}. \tag{40}$$

28. If we now substitute the values of a and b in equation (25), we find that $\omega^4 Q^2$, which occurs in equation (28), is given by

$$\omega^4 Q^2 = \frac{[X - Y]^2 + 4\kappa^2\omega^{4}\tau^{4}\mu^2 (1 - \mu)^4}{m^2 [\mu^2 (\pi^2 + \kappa^2)^2 + 4\mu^2\kappa^2\omega^2\tau^2 + \mu^2\omega^4\tau^4 - 2\mu^2\omega^2\tau^2 (\pi^2 + \kappa^2)]^2}, \tag{41}$$

where

$$X \equiv \mu\tau^2\omega^2 (\pi^2 + \kappa^2) (1 + \mu^2 + 6\mu),$$

and

$$Y \equiv 16\kappa^2\omega^2\tau^2\mu^2 + 4\mu^2 (\pi^2 + \kappa^2)^2 + \mu\omega^4\tau^4 (1 + \mu)^2.$$

Let us place

$$\delta \equiv \omega\tau \equiv \omega/\nu_o, \tag{42}$$

where ν_o is the natural frequency of the oscillator.

Equation (41) may then be written, after some simplifications, as

$$\omega^4 Q^2 = \frac{\{M (\pi^2 + \kappa^2) - S\}^2 + 4\kappa^2\delta^2 (1 - \mu)^4}{m^2\mu^2 [(\pi^2 + \kappa^2)^2 + \delta^4 + 2\delta^2 (\kappa^2 - \pi^2)]^2}, \tag{43}$$

where

$$M \equiv \delta^2 (1 + \mu^2 + 6\mu) - 4\mu (\pi^2 + \kappa^2),$$
$$S \equiv 16\mu\kappa^2\delta^2 + \delta^4 (1 + \mu)^2.$$

29. If κ and δ are small, we find, to a sufficient degree of approximation, that

$$\frac{\omega^4 Q^2}{16} = \frac{1}{m^2} (1 - g_1\delta^2 + g_2\delta^4), \tag{44}$$

where

$$\left.\begin{aligned} g_1 &\equiv \frac{1 + \mu^2 - 2\mu}{2\mu\pi^2}, \\ g_2 &\equiv \frac{\mu^4 - 12\mu^3 + 22\mu^2 - 12\mu + 1}{16\mu^2\pi^4}, \end{aligned}\right\} \tag{45}$$

and

$$g_2\delta^4 << g_1\delta^2 < 1.$$

30. Thus, by virtue of equations (27) and (44), the ratio of the intensity of the scattered radiation at a point distant r from the centre of the electron and making an angle θ with the incident beam, to that of the incident radiation is given by

$$v = \frac{e^4(1+\cos^2\theta)}{2m^2r^2c^4}(1 - g_1\delta^2 + g_2\delta^4), \tag{46}$$

and the coefficient of scattering per electron, σ_1, is seen, by virtue of equations (28) and (44), to be given by

$$\sigma_1 = \frac{8\pi e^4}{3m^2c^4}(1 - g_1\delta^2 + g_2\delta^4). \tag{47}$$

If there are N atoms per c.c. and p electrons per atom, the coefficient of scattering for the Np electrons is

$$\sigma = \frac{8\pi e^4 Np}{3m^2c^4}(1 - g_1\delta^2 + g_2\delta^4). \tag{48}$$

31. It is evident that the coefficient of scattering given by (48) reduces to the Thomsonian one when δ is negligibly small, that is, when the wave-length, λ, of the incident radiation is large. When, however, λ is not sufficiently large, δ is not negligible and according to (48) there should be a diminution in the scattering coefficient when short waves are used. This decrease in σ will depend on the values assumed for μ and δ, and by this two-fold choice of arbitrary constants one should be able to find a theoretical curve for the variation in σ with the wave-length which agrees, as well as may be, with the experimental results. If we divide both sides of equation (48) by

$$\sigma_c = \frac{8\pi e^4 Np}{3m^2c^4},$$

which is the value of σ according to the classical theory, we obtain the quantity

$$F \equiv (1 - g_1\delta^2 + g_2\delta^4), \tag{49}$$

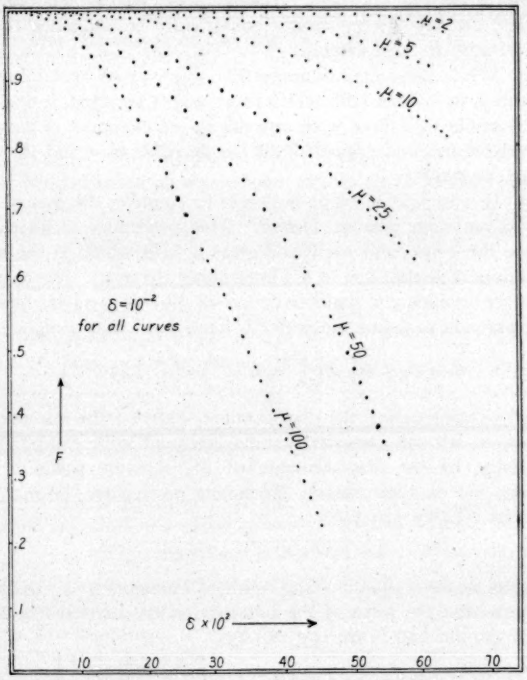
which is the factor by which Thomson's expression for σ is to be multiplied in order to find the observed scattering coefficient. A series of curves for F, using different values of μ, are given in Fig. 6.

32. It will be observed that while the present theory gives a scattering coefficient depending on the wave-length of the incident radiation, it predicts a distribution of the intensity of the scattered beam which is symmetrical with respect to the radiating

plate. This, as we have already remarked, is not borne out by experiment.

33. We have seen that the observed decrease in the total scattering may be explained in terms of the properties of the

FIG. 6.



electron itself. In the endeavor to explain the observed asymmetry also in terms of the properties of the electron, the writer was led to consider various models of the electron and certain cumulative effects. Among these may be mentioned (1) the "shielding effect" (*i.e.*, the fact that the force acting on any electron will not merely be that due to the incident beam, but, in addition, there will

be a contribution from all other electrons in the neighborhood of the one considered, due to their motion); (2) the effect of the magnetic force of the incident beam; (3) an electron consisting of a doublet and radiating only in a certain direction; and (4) an electron consisting of two parts having a doublet superimposed upon one of them. Investigation has shown that the fore and aft asymmetry cannot consistently be explained on the basis of any of these effects.

34. While there may be more than one way of explaining the diminution in σ when sufficiently short waves are used, it does not seem possible to adhere with any degree of closeness to classical electrodynamics and explain both the decrease in σ and the fore and aft asymmetry.

35. It was next thought desirable to combine the theory outlined above with that of Debye. This procedure is legitimate because the compound oscillator gives a field which is the same function of θ as that due to a Thomsonian electron. The changes which are necessary to make in equations (3) to (13) are obvious if we take care to define the vector U such that [9] its z-component is

$$U_z \equiv U_{z_1} + U_{z_2} = \left(\frac{e}{2r}\ \xi_{n_1}\varepsilon^{-ikr}\right) + \left(\frac{e}{2r}\ \xi_{n_2}\varepsilon^{-ikr}\right).$$

Instead of considering the displacement (ξ_n) of the nth electron as a whole, we will consider this displacement to be compounded, vectorially, of the displacements of the separate parts of this electron. It is then merely necessary to replace (e/mc^2) in equations (3) to (7) by

$$k^2\sqrt{(4e^2/m^2\omega^4)\,(1 - g_1\delta^2 + g_2\delta^4)},$$

where, as above, $k = \omega/c$. The result of introducing the indicated change is that the ratio of the intensity of the scattered beam to that of the incident beam is given by

$$V = \frac{Npe^2g^2k}{4R^2}\left(\frac{1+\cos^2\theta}{2}\right)\sum_{n=o}^{p-1}\frac{\sin\left[4\,ka\sin\frac{n\pi}{p}\sin\frac{\theta}{2}\right]}{\left[4\,ka\sin\frac{n\pi}{p}\sin\frac{\theta}{2}\right]} \tag{50}$$

where

$$\frac{g^2}{4} \equiv e^2\,(1 - g_1\delta^2 + g_2\delta^4)/m^2\omega^4,$$

and the other symbols have the same significance as before.

36. The scattering coefficient may be obtained by integration and comes out to be

$$\sigma = \frac{Npe^4\,(1 - g_1\delta^2 + g_2\delta^4)}{m^2c^4}\int VR^2 d\Omega\,. \tag{51}$$

This expression leads to two special cases corresponding to those already considered above.

Case 1: If the wave-length, λ, of the incident beam is much greater than the radius of the ring (a), then

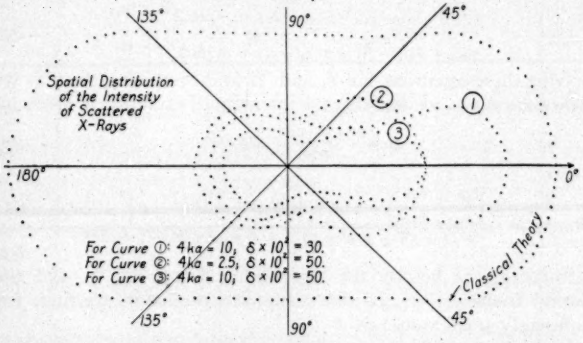
$$\sigma = \frac{8\pi e^4}{3m^2c^4}\,(1 - g_1\delta^2 + g_2\delta^4)\,Np^2\,. \tag{52}$$

Case 2: When $\lambda << a$, then

$$\sigma = \frac{8\pi e^4}{3m^2c^4}\,(1 - g_1\delta^2 + g_2\delta^4)\,Np\,. \tag{53}$$

It will be observed that equations (50) to (53) predict "excess

FIG. 7.



scattering," fore and aft asymmetry, and a decrease in the scattering coefficient.

37. The function

$$\phi_{p,\delta}(j) \equiv (1 - g_1\delta^2 + g_2\delta^4)\,\frac{1}{p}\sum_{n=0}^{p-1}\frac{\sin\left[j\sin\dfrac{n\pi}{p}\right]}{\left[j\sin\dfrac{n\pi}{p}\right]}\,, \tag{54}$$

where $j = 4ka\,\sin\frac{\theta}{2}$ is displayed graphically in Fig. 3 for different values of p, j, and δ. In Fig. 7 are given the theoretical intensity-distribution curves according to the classical theory and also according to the modified Debye theory for different values

of δ and $4ka$. Fig. 5 gives the experimental distribution curve for Al as found by Crowther [13] and the theoretical distribution curves for hydrogen according to Debye's theory, modified and unmodified. It will be seen that, although the curves are plotted for different substances, the experimental and theoretical curves are, at any rate as regards their form, in excellent agreement.

38. It is desirable to record here that instead of assuming that the electron consists of two parts, each having a charge of $(-e/2)$, we might have assumed that the two parts were unequally charged, *e.g.*, one part could have a charge of $-2e$ and the other part a charge of $+e$. Such an assumption has, in fact, been made, and it will be of interest to state briefly the differences between this oscillator and the one already discussed.

39. Using the same notation as before, the equations of motion of this oscillator are assumed to be:

$$\left.\begin{aligned} m_1\ddot{\xi}_1 + b(\dot{\xi}_1 - \dot{\xi}_2) + a(\xi_1 - \xi_2) &= -2eE_0\varepsilon^{i(\omega t - kx)}, \\ m_2\ddot{\xi}_2 + b(\dot{\xi}_2 - \dot{\xi}_1) + a(\xi_2 - \xi_1) &= +eE_0\varepsilon^{i(\omega t - kx)}. \end{aligned}\right\} \tag{55}$$

Solving these equations for $\ddot{\xi}_1$ and $\ddot{\xi}_2$ and combining them as we have done above, we find that the intensity of radiation is given by

$$I_\theta = \frac{e^4(1+\cos^2\theta)}{2r^2c^3}\,\omega^4 E_0^2 P^2, \tag{56}$$

where

$$P^2 = \frac{\left\{(1+\mu)\left(\frac{b^2}{m_2} + \frac{a^2}{m_2\omega^2}\right) - a(\mu^2+6\mu+4) + m_1\omega^2(\mu+4)\right\}^2 + b^2\omega^2(\mu+2)^4}{[a^2(1+\mu^2) + m_1\omega^4 - 2a(1+\mu)m_1\omega^2 + b^2\omega^2(1+\mu)^2]^2}. \tag{57}$$

Introducing, as before, the logarithmic decrement, κ, and the natural frequency, ν_0, of this compound oscillator, we find, for sufficiently small values of κ and δ,

$$\omega^4 P^2 = \frac{1}{m^2}\left[1 - \left(\frac{2\mu^2+8\mu+8}{\mu\pi^2}\right)\delta^2 + \left(\frac{\mu^4+6\mu^3-8\mu^2+24\mu+16}{\mu^2\pi^4}\right)\delta^4\right],$$

where, as above, $\delta \equiv \omega/\nu_0$.

40. The scattering coefficient per electron is given by

$$\sigma_1 = \frac{8\pi e^4}{3m^2c^4}(1 - h_1\delta^2 + h_2\delta^4), \tag{58}$$

where

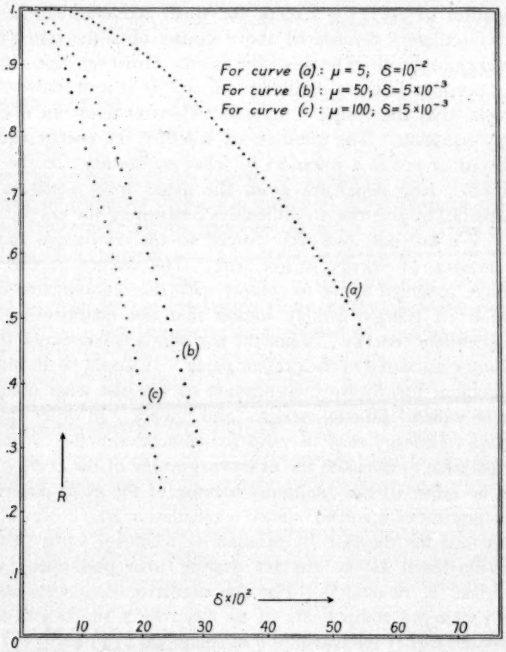
$$\left.\begin{aligned} h_1 &\equiv \frac{2\mu^2+8\mu+8}{\mu\pi^2}, \\ \text{and} \qquad h_2 &\equiv \frac{\mu^4+6\mu^3-8\mu^2+24\mu+16}{\mu^2\pi^4} \end{aligned}\right\} \tag{59}$$

If we divide this expression by the value of σ given by the classical theory, we obtain the factor

$$R \equiv [1 - h_1\delta^2 + h_2\delta^4]. \tag{60}$$

The values of R are plotted against δ for different values of μ and δ in Fig. 8. Here again it is seen that the diminution in

FIG. 8.



the scattering coefficient may be accounted for; but, as in the previous case, a theory based on a scattering unit of this kind cannot explain the dissymmetry of the spatial distribution of the intensity of the scattered radiation.

41. Upon inspection of equations (55) to (58) and (19) to (53) it will be evident that the combination of the present

modified electron with Debye's theory would give results similar to those resulting from the combination already discussed.

THE CONSERVATION OF ENERGY.

42. We shall now consider briefly the difficulties concerning the energy equation which were mentioned in paragraph 23. The term $b(\dot{\xi}_1 - \dot{\xi}_2)$ is akin to a frictional term and gives rise to a dissipation of energy. Except for small accelerations the compound oscillators considered above do not obey the conservation of energy in the strict sense of the word. However, this difficulty is not peculiar to these oscillators alone. It is well known,[15] for example, that the simple Lorentzian electron does not obey the energy equation. The question of whether the energy equation is obeyed or not is a question of what we assume for the force equation. Any departure from the usual force equation may be expected to give rise to difficulties concerning the energy equation. We are not, however, forced to the conclusion that the conservation of energy is not true. The failure of the force equation, assumed above to comply with the requirements of the equation of energy, simply means that the quantity we have usually called "energy" is not the one which is conserved for the oscillators discussed in the present paper. It would be an interesting problem for future investigation to see just what quantities must be called "kinetic energy" and "work" in order that the equation of energy shall be valid for all accelerations. It has not been possible to calculate the exact magnitude of the energy dissipated by either of the oscillators because of the great uncertainty of the quantities involved. Such a calculation has, however, been carried out for the case of resonance—whatever value this may have, inasmuch as we are *not* dealing with phenomena which take place at resonance. For the oscillator consisting of two equally charged components of an electron, a simple calculation has shown that if the constant b in equations (14) and (15) is of the order 10^{-3}, and μ of the order 10, the energy per square centimetre of the incident beam which is dissipated is of the order of .5 per cent. As far as any experimental evidence is concerned there may actually be a dissipation of energy in the atom, and if so, it may take place in some such fashion as that indicated in this paper.

43. It will be instructive, following Schott, to compare, in tabulated form, the various theories proposed for explaining the scattering of X-rays.

Theory.	Max. total scattering (long λ's) is proportional to	Min. total scattering (short λ's) is proportional to	Max. asymmetry.
Thomson's.....	Np	Np	0
Electron ring	$N\Sigma p_i^2$	Np	Increases with p.
Ring electron	Np	0	Always large.
Present work combined with electron ring	$N\Sigma p_i^2$	0	Increases somewhat with p.

In the above, N is the number of atoms per c.c., and p_i is the number of electrons in the ith ring, and Σp_i^2 is the summation of p_i^2 taken over all the rings in the atom if the latter consists of more than one ring.

44. To review, we may say that in the scattering of X-rays of short wave-lengths by the lighter elements the observed scattering coefficient is much less than that predicted by the classical theory. It has thus far not been possible to explain satisfactorily this diminution in the scattering coefficient on the basis of classical electrodynamics. The work herein presented shows how, by a slight modification of the ordinary equation of motion of an electron, we may account for the observed decrease in the scattering coefficient; and that by considering each electron as a sort of compound oscillator and making use of Debye's theory we may also account for the observed asymmetrical distribution of the intensity of the scattered beam. In this investigation, the quest was not for perfect agreement between theoretical predictions and the experimental results on one or two elements, as our experimental knowledge of the scattering of X-rays does not at all warrant such a procedure. The aim has rather been to discuss the possibility of modifying the classical theory so that it may lead to a reasonable explanation of the observed facts.

The writer deems it a pleasure to acknowledge his indebtedness to Prof. W. F. G. Swann, who suggested the problem, for criticisms and suggestions.

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Results from Magnetic East-West Paths around the Earth. W. J. PETERS. (*Terrestrial Mag.*, Sept., 1923.)—"In the preliminary analysis of the earth's magnetic field for 1922, by Dr. Louis A. Bauer, it was found necessary to ascertain how an east-west path around the earth would close that was everywhere throughout its length perpendicular to the compass needle, or to the horizontal component of the earth's magnetic field." Eleven such lines were drawn by the use of isogonic charts. Five of these were plotted both by the author and by another worker. "The greatest difference in their independent determinations of the lack of closure is less than eighteen statute miles." The most northerly of the lines after leaving a point near Greenwich goes in a northeasterly direction to eastern Poland, where it runs parallel to the equator. It thence turns to the southeast, and in Mongolia again becomes parallel to the equator. Its course then bears to the northeast, once more to become due east and west somewhat west of the Aleutian Islands. Next a long and decided sweep to the southeast follows. The line enters the United States near San Francisco, runs to Savannah, where it again parallels the equator, and in the 82 degrees of longitude between this city and Greenwich mounts about 20 degrees toward the north.

The other similarly constructed lines are not greatly different in shape. It should, however, be noted that the first and eleventh lines in the longitude of Liberia are about 99 degrees of longitude apart, whereas in the longitude of Manila the interval is only two-thirds as much. The line starting on the Greenwich meridian and 50° north latitude shows four points of inflection in its course, while the line with its origin on the same meridian but 50° south latitude has only two. G. F. S.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

GRAVITATION ANISOTROPY IN CRYSTALS.[1]

By Paul R. Heyl.

[ABSTRACT.]

EINSTEIN'S theory of gravitation is based upon a fundamental postulate which he calls the principle of equivalence, and which asserts that gravitation and inertia are identical in nature, and hence indistinguishable. This, if true, is of the greatest theoretical importance, for gravitation has steadily refused to show any kinship to other physical phenomena.

Einstein's justification for this postulate was found in the gravity pendulum experiments of Newton and Bessel, and the torsion pendulum experiments of Eötvös, the results of which established to a high degree of precision (about 1 part in 200,000,000) that the inert mass and the gravitational mass of a body were proportional, or, in other words, that gravitation is independent of the nature of the matter acted upon.

A still more delicate test of this postulate is possible in a crystal of one of the non-isometric systems; for in such a crystal every known physical property (except inertia, and possibly weight) varies with the axial direction in the crystal. It is therefore an interesting question whether, in such a crystal, gravitation will be found to align itself with inertia or will show some variability which will classify it with the great majority of physical phenomena. If, for example, such a crystal be weighed in different axial orientations with respect to the earth (which may be done with great precision) and any difference in weight be found in the different positions, Einstein would be wrong.

To test this point, crystals weighing a kilogram or more were thus weighed, the specimens covering all five non-isometric crystalline systems. The precision reached, in nearly every case, was one part in a billion (10^9). To this degree of precision no difference in weight was detected; the results have failed to prove Einstein wrong.

* Communicated by the Director.

[1] Scientific Papers, No. 482, price ten cents.

Incidentally, this work has shown the practical possibility of using the gravity balance to a precision of 1 part in 10^9 even when the swing of the beam must be stopped and the object turned through a considerable angle. This improved precision was reached by the use of the almost neglected Poynting clamp, which permits the arrest of the beam and the changing of the load without raising the knife-edge or altering the state of flexure of the beam.

A precision of about the same order was attained by Majorana in 1920. In this work it was not necessary to arrest the beam or to touch the load. The next best record (in work of a somewhat different kind) at the International Bureau of Weights and Measures is 7 parts in 10^9.

THE PREPARATION AND PROPERTIES OF PURE IRON ALLOYS: IV. DETERMINATION OF THE CRITICAL RANGES OF PURE IRON-CARBON ALLOYS BY THE THERMO-ELECTRIC METHOD.[3]

By J. F. T. Berliner.

[ABSTRACT.]

THE object of this investigation was to determine the critical ranges of pure iron-carbon alloys by means of the thermo-electric method.

The material used was a very pure series of iron-carbon alloys prepared at the Bureau of Standards. For the purpose of this work the steel specimens were cold drawn to wires of about 0.07 cm. diameter.

The method used was practically the same as that employed by Burgess and Scott in 1916 (see B. S. Sci. Paper No. 296) which was a modification of that of Le Chatelier's. The steel wire specimen had each of its ends welded to the hot junction of a platinum; platinum-rhodium thermocouple. The thermocouple gave the temperature at the ends of the wire, while the platinum wires of each thermocouple were used as leads from the specimen and gave the e.m.f. reading. The thermo-electric power was measured at intervals of 2° C. from room temperature up to 1150° C. The temperature difference between the ends of the wire specimen was usually less than 12° C.

[3] Scientific Papers, No. 484, price five cents.

This method affords a very accurate means of studying the critical ranges of iron and steel. The results show that the temperature of the A_2 transformation (768° C.) is constant for heating and cooling up to a concentration of 0.45 per cent. carbon. When this carbon content was exceeded, the A_2 transformation could not be detected. This is somewhat in disagreement with the magnetic determinations of the A_2 transformation by Honda, whose investigation seemed to indicate that the A_2 transformation extends completely across the iron-iron carbide equilibrium diagram.

The temperatures of the A_3 and A_1 transformations are in close agreement with those obtained by other methods. An important feature is that the upper end of the A_3 transformation is sharp. This is not the case when the determination is made by other methods.

THE NICK-BEND TEST FOR WROUGHT IRON.[3]

By Henry S. Rawdon and Samuel Epstein.

[ABSTRACT.]

THE "nick-bend" test is a requirement included in most current specifications for wrought iron, the character of the fracture being the criterion by which the material is judged. This test has, without doubt, been developed from the blacksmith's method of breaking a metal bar by notching and then completing the break by a few sharp hammer blows. In American specifications for wrought iron, the "nick-bend" clause allows very considerable latitude in carrying out the test; the nicked specimen may be broken "either by pressure or by blows."

This investigation was carried out to show how the results may vary with the method used and was based upon a series of twelve grades of iron manufactured to meet certain A. S. T. M. specifications. All specimens were notched in the same manner, the testing machine and a special chisel being used. The specimens were fractured in different ways, varying from simple transverse bending to a single-blow, impact test.

The results indicated that the rate at which the fracture is produced is of fundamental importance in determining its character. Crystalline areas were found in the fractures of all the bars

[3] Technologic Papers, No. 252, price ten cents.

broken by severe impact. The same materials, however, when broken by a slowly applied stress, as is permitted by most specifications, gave a fibrous break. The character of the iron also affects the results. Wrought iron containing small slag threads which are uniformly distributed show larger crystalline areas in the fractures after the nick-bend test than do irons that are not so uniform. Open-hearth iron, which contains no slag threads whatever, practically always gives a crystalline break, no matter how the break is made; while wrought iron with prominent slag streaks is apt to split and sliver, thus giving a fibrous appearance.

The test is popularly supposed to indicate whether the iron contains "steel." The results obtained show definitely that "crystallinity" is not necessarily indicative of steel, although, on the other hand, steel streaks, if present, do appear to aid in producing a crystalline break; nor can the test be used for indicating whether the iron is high in phosphorus or not.

Only the very coarsely granular, sparkling areas in a fracture should be considered. Many fractures contain portions which, by the inexperienced man, may be mistaken for "crystallinity." There were two distinct types of crystalline areas—one on the tension side of the bar, the other in that part subjected to compression. That on the compression side of the specimen is caused by the flattening of the grains of the material by compression during the progress of the test. Such areas should not be considered as indicating the same property of the material that the "tension" areas do.

A series of notched bar impact tests of open-hearth iron (ferrite) was carried out for the purpose of showing how "crystallinity" is related to grain structure. The results indicated that an increase in grain-size, such as occurs upon annealing, is not necessarily accompanied by low impact resistance. Uniformity in shape and size of the enlarged ferrite grains is usually indicative of a transcrystalline fracture ("crystallinity"), and a low resistance to impact. Similar material treated in the same manner, but showing a somewhat irregular "interlocking" grain structure, gave a relatively high impact resistance and broke with a fibrous fracture.

In general, the results obtained indicate strongly the need for defining more strictly the conditions for carrying out this test than is the case in most American specifications. In particular,

the manner of application of stress should be definitely stated. The nick-bend test as often carried out is an approximation of the notched-bar impact test.

SOME NEW THERMO-ELECTRICAL AND ACTINO-ELECTRICAL PROPERTIES OF MOLYBDENITE.[4]

By W. W. Coblentz.

[ABSTRACT.]

WHEN molybdenite is heated by thermal conduction or exposed to thermal radiation it exhibits hitherto undescribed thermo-electric, actino-electric, and photo-electric properties.

Thermo-electrical Properties.—The first part of this paper gives data on the thermo-electric power of samples of molybdenite obtained from various localities. The observed values of thermo-electric power against copper range from about +700 microvolts, through practically zero thermal e.m.f. to −1040 microvolts. Evidently standard thermo-electric data cannot be specified.

Actino-electrical Properties.—A new actino-electric phenomenon is described, consisting of an e.m.f. which is manifested in spots or loci of some samples of molybdenite, when exposed to thermal radiation of short wave-lengths. The polarity of the actino-electric e.m.f. may be positive or negative, in spots which are separated by only 0.5 to 1.5 mm.

The spectral actino-electric effect may be photopositive or photonegative, depending upon the wave-length of the thermal radiation stimulus. The maximum actino-electrical e.m.f. is usually produced by radiation stimuli of wave-lengths between 0.65 and 0.9μ, and no effect seems to be produced by radiation stimuli of wave-length greater than 1μ.

The time for attaining the maximum of the actino-electric response appears to be instantaneous irrespective of the wave-length of the exciting radiation.

Photo-electrical Properties.—The photo-electrical effect (described principally in B. S. Sci. Pap. Nos. 338 and 398) is manifested as a change in resistance when the substance is subjected to an impressed potential and exposed to light. This change in resistance may be photopositive or photonegative, depending upon the wave-length of the exciting radiation. The photo-electrical

[4] Scientific Papers, No. 486, price ten cents.

reaction seems to be produced by radiation of wave-lengths from 0.3μ to 2μ.

The time for attaining the maximum of the photo-electrical responses is a function of the wave-length of the exciting radiation, requiring from one to several minutes, and twice that time for recovery after exposure. However, in the typical class of molybdenite having a low resistance, a low photo-electrical sensitivity but a high actino-electrical sensitivity, the spectro-photo-electrical reaction appears to be instantaneous, irrespective of the wave-length.

Data are given on the effect of temperature, of the intensity of the exciting radiation, etc. In general, these factors have no marked effect upon the spectral actino-electric reaction, or are just the reverse of the effect upon the spectrophoto-electric reaction.

When there is a close coincidence of the spots exhibiting photo-electrical and actino-electrical sensitivity, the photo-electric current is amplified or weakened, depending upon the direction of the impressed battery through the crystal. This selective response produces a difference of 1.5 to 2.5 times between the maximum and the minimum photo-electric effect.

On the Measurement of the Surface Tension of a Small Quantity of Liquid. A. FERGUSON. (*Proc. Phys. Soc.*, Dec. 15, 1923.)—In a vertical capillary tube is put a little of the liquid. Pressure is applied to upper surface of the liquid. By adjusting this the meniscus at the lower and open end of the tube is made flat. The surface tension can then be calculated from the applied pressure, the radius of the tube, the density of the liquid and the distances between its upper and lower surfaces. When applied to water and benzene this method gave excellent results. The planeness of the lower meniscus was judged by examining with a lens the image of the filament of a lamp reflected by it. In one case only .00096 cubic centimetre of liquid was used for the determination. When such a small volume is employed its density need not be very accurately known.

If there are in the vertical tube two liquids in contact, the surface tension at their interface may be got by applying pressure until the meniscus between them becomes flat. Other interesting uses of the method are given.

This paper deserves the attention of those who need to devise laboratory experiments on surface tension. G. F. S.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

THE DISPERSITY OF THE SILVER HALIDES IN RELATION TO THEIR PHOTOGRAPHIC PROPERTIES.[1]

By S. E. Sheppard.

A REVIEW of the more recent work on the silver halides and their photographic characteristics. In this paper are discussed the form and dispersity of the silver halides; the factors influencing their dispersity; the two types of reduction of dispersity; the relation of dispersity to light sensitivity, particularly with respect to negative emulsions; the distribution of grain-size and its possible influence on photographic properties; the origin of the distribution; photographic characteristic curves and the empirical properties of photographic plates; the theory of the density-exposure function; the work on one-grain-layer plates, and on individual grains; random distribution of sensitivity of the grains due to the inherent variation of sensitivity, and to conditions in emulsification, and the non-continuous quantum distribution of energy in light. Silberstein's development of the latter theory is considered in detail.

THERMAL CHARACTERISTICS OF MOTION PICTURE FILM.[2]

By L. A. Jones and E. E. Richardson.

THIS article contains a description of two calorimeters (liquid and air); of the method which was employed in the standardization of the latter; of the procedure for measuring the specific heat of motion picture film; and of a method of measuring the change in temperature of the film during the process of projection. The water equivalent of the air calorimeter was found to be 961 ± 6 gram-calories. The specific heats of film on cellulose nitrate base,

* Communicated by the Director.

[1] Communication No. 188 from the Research Laboratory of the Eastman Kodak Company. Published in "Colloid Symposium Monograph," University of Wisconsin, 1923, p. 346.

[2] Communication No. 196 from the Research Laboratory of the Eastman Kodak Company. Published in *Trans. Soc. Mot. Pict. Eng.*, 17, 86, 1923.

with the silver image absent, and film having a uniform optical density of 3.46, were found to be 0.434 and 0.427 gram-calorie, respectively. The difference between "clear" film and film which transmitted 0.1 per cent. of visible radiation is, therefore, only 0.007, which is equivalent to approximately 2 per cent. Thus the specific heat of motion picture film is practically independent of the density of the picture. For cellulose acetate base this value was found to be 0.482 gram-calorie. Using a standard projector, adjusted as near as possible to practical conditions, film containing practically no developed silver, film having an average optical density of 0.56, and film having a uniform density of 3.46 absorbed 6.0, 12.9, and 13.3 gram-calories, respectively. In case the projecting room is at a temperature of 80° F., the speed of projection 70 feet per minute, and the arc current 100 amperes, the temperature rise of the film is 29.5° F. Thus the film would leave the gate at a temperature of 110° F., approximately.

The Near Infra-red Absorption Spectra of Some Organic Liquids. J. W. ELLIS. (*Phys. Rev.*, Jan., 1924.)—"While rocksalt has proved to be the best prism material when investigations into the far infra-red region of the spectrum are desired, since it transmits as far as about 15μ, in studying the near infra-red region, prisms with a high dispersive power, such as quartz, glass or carbon bisulphide may be used." The radiation was passed through two flint glass prisms in series, and then back a second time through them, so that the dispersion was four or five times that obtained with a single 60° rocksalt prism. After completing its journey through the prisms the radiation fell on a thermopile joined to a galvanometer. A beam of light reflected from the mirror of this instrument fell upon sensitive paper wrapped about a cylinder that was rotated by the same mechanism that turned one of the prisms. By the rotation of the latter all wave-lengths from .589μ to 2.7μ were made to fall in turn on the thermopile. The rotation of the cylinder of paper combined with the motion of the spot of light perpendicular to its axis governed by the galvanometer mirror, recorded the absorption spectrum of any liquid traversed by the light. "Some thirty substances have been examined with the self-recording photographic device. They include chloroform, bromoform, certain ethyl, methyl, propyl, butyl, ethylene and methylene halides as well as some simple hydrocarbons, such as benzene, toluene, pentane, hexane, etc. . . . Striking similarities among these spectra are evident. Besides the bands common to all in the neighborhood of 1.17μ, 1.38μ, and 1.7μ, in many of these compounds bands near .9μ and 1.0μ appear." These are attributed to the C-H bond.

G. F. S.

NOTES FROM THE U. S. BUREAU OF CHEMISTRY.*

DIPYRIDYLS FROM PYRIDINE.[1]

By C. R. Smith.

[ABSTRACT.]

α, α, β, β, β, γ, and γ, γ dipyridyls have been produced by a new method involving the interaction of sodium, pyridine, and dry oxygen or air.

Some of the dipyridyls are toxic to insects and the new method supplies an economic procedure for preparing them for study and possibly for commercial production.

β, γ dipyridyl, the only one of the six possible isomers not previously reported, has been isolated and some of its properties determined.

The proportions in which the various dipyridyls are produced depend principally on the time and temperature of the preliminary digestion of the sodium dipyridine first produced in the excess of pyridine before the oxidation is begun.

EXCRETIONS FROM LEAVES AS A FACTOR IN ARSENICAL INJURY TO PLANTS.[2]

By C. M. Smith.

[ABSTRACT.]

COMMERCIAL calcium arsenate conforming to all our present standards occasionally causes severe foliage injury when dusted upon cotton plants for the control of the cotton boll weevil. The only explanation suggested was "peculiar atmospheric conditions." It has now been found that the dew which forms upon Upland cotton plants is in general alkaline to phenolphthalein, and contains mineral salts to the extent of about 1000 parts per million. These salts are mainly calcium and magnesium bicarbonates and are probably taken up from the plant by osmosis. Such salts are capable of decomposing some commercial calcium arsenates with

* Communicated by the Chief of the Bureau.

[1] Published in *J. Am. Chem. Soc.*, **46** (Feb., 1924) : 414.

[2] Published in *J. Agr. Research*, **26** (Oct. 27, 1923) : 191.

the liberation of soluble arsenic. One experiment with the dew itself showed 8.7 per cent. soluble As_2O_5 in a sample which gave only 0.08 per cent. with distilled water. These facts suggest that the reaction of dew with arsenical sprays or dusts may at times be the cause of foliage injury to cotton and other plants.

Free Convection of Heat in Liquids. R. A. NELSON. (*Phys. Rev.*, Jan., 1924.)—A wire surrounded by liquid was heated by the passage of an electric current. The convection current thereby caused was photographed both by the parallel light and by the schlieren method. The latter gave more details of the immediate vicinity of the wire. In the case of all liquids the ascending current forms a sort of neck a few millimetres above the wire. In water the formation of the current was studied by taking photographs ½, 1 and 1½ seconds after the turning on of the electric current. In the earliest of these the wire is seen surrounded almost, but not quite, by a layer of heated water. Though this layer extends below the wire, there is a gap in it just under the latter. After one second the wire is entirely surrounded by the warmed liquid, which, however, extends much farther above the wire than below it. "The appearance of the image of the wire tends to confirm Langmuir's suggestion of a stationary film on the surface. If the molecules of the liquid received heat only as they came in contact or very close to the wire, the heated liquid would not be seen at the lower side of the wire." Other observations are held to substantiate Langmuir's theory. G. F. S.

The Fluorescence of Certain Organic Compounds. E. BAYLE and R. FABRE. (*Comptes Rendus*, Feb. 11, 1924.)—After exposure to ultra-violet light novacaine fluoresces strongly while cocaine and stovaine under the same conditions remain dark. From a mixture of these substances that has been in ultra-violet light, it is possible to pick out the crystals of novacaine with forceps. In like manner when a mixture of salicylate of soda, talc and borax has been subjected to very short wave-lengths of light, the first of these substances glows with strong violet light, but the others are dark. The authors have been led by such practical applications of fluorescence as these to make a study of a number of organic compounds of somewhat common use. They were exposed to the light of the mercury ray of wave-length 3650 Å. Benzene and phenol showed no fluorescence at all. Most of the forty substances emitted light of some bluish tint whose wave-length was about 1000 Ångströms longer than that of the exciting light. The radiation from only three contained any green light. G. F. S.

NOTES FROM THE U. S. BUREAU OF MINES.*

ANALYTICAL METHODS FOR TITANIUM.

By J. W. Marden.

METALLIC titanium has properties that restrict its use. It is white, but cannot be readily forged; it burns in oxygen to the oxide, and reacts directly with nitrogen.

Variable quantities of titanium occur in granite, gneiss, mica slate, syenite rocks, and sometimes in granular limestone and dolomite, as well as in magnetite, hematite, monazite, zirconium ores, and auriferous sands. Ilmenite is often present in deposits of magnetite. In recent years the commercial importance of titanium has increased and methods for its detection and analysis have, therefore, been developed.

The most important use of titanium is in making the alloy, ferro-titanium, which is employed chiefly as a deoxidizer of iron and steel. Paints for protective coatings on iron and steel are made by mixing 10 to 15 per cent. asphalt with rutile and thinning with turpentine. Titanium compounds are also used as dyes, mordants, bleaching cloth, and in pigments for glazing china.

Methods of analysis for titanium, which have been thoroughly tested by the Bureau of Mines, are outlined in Bureau of Mines Bulletin 212, "Analytical Methods for Certain Metals."

CO HAZARDS FROM HOUSE HEATERS BURNING NATURAL GAS.

By G. W. Jones.

IN its investigations relating to CO in atmospheres in mines, tunnels, and around metallurgical furnaces, the Bureau of Mines has studied hazards from the liberation of CO from house heaters and stoves burning natural gas. Six common types of heaters were tested as supplied by the manufacturers without readjustment, in a closed room without ventilation. In most of the tests not enough CO was liberated to cause any alarm as to the use of natural-gas heaters, or to necessitate any radical changes in

* Communicated by the Director.

their operation. Only under abnormal conditions, caused by faulty construction, excessive gas flow, or improper adjustment of burner, did the CO become dangerous.

As a safety measure, a heater should not be allowed to burn in a closed room in which persons sleep, especially if the heater burns much gas and has no flue, unless one or more windows are open to supply plenty of fresh air. With this precaution, there should be no loss of life by CO poisoning.

Further details are presented in Technical Paper 337 of the Bureau of Mines.

CENTRAL DISTRICT BITUMINOUS COALS AS WATER-GAS GENERATOR FUEL.

By W. W. Odell and W. A. Dunkley.

A STUDY of central district coals as generator fuel for water-gas plants in Illinois has been completed by the Bureau of Mines, in coöperation with the University of Illinois and the State Geological Survey. About two-thirds of the manufactured gas used in Illinois is carburetted water, and the leading generator fuel is coke from eastern coals. The object of the investigation is to promote the use of Illinois coals for gas-making.

The practice already developed was studied, difficulties and what had been done to meet them, coals were tested to find the ones best suited to the purpose.

It was found that a number of plants are operating successfully with bituminous coal, at a saving of as much as eight cents per 1000 feet of gas. While present-day water-gas plants have mostly been designed for fuel low in volatile and high in fixed carbon, an operating method was worked out, whereby some of the difficulties met in using high-volatile fuel can be overcome. As yet no specially designed set has been worked out. Details of the tests are presented in Bulletin 203 of the Bureau.

REACTION BETWEEN ZINC OXIDE AND OXYGEN.

By Carl E. Baumgarten and Charles G. Maier.

IN experimental work on the chemistry of zinc reduction conducted by the University of Utah in coöperation with the Bureau of Mines, it was found that oxygen follows the gas law when heated in contact with zinc oxide (up to 480°). The change in

color of zinc oxide from white to yellow, when heated, is not due, as some have believed, to the formation of zinc peroxide, and the peroxide cannot be made by heating zinc oxide in oxygen up to 480°, the upper limit of the tests. The color change begins at about 413° C. and the oxide gradually reaches a bright yellow at about 476°. Pumping out the air to 1 mm. pressure has no apparent effect on the color change.

HYDROMETALLURGY OF OXIDIZED ZINC ORES.

By C. G. Maier, G. L. Oldright, and G. W. Kuener.

The Bureau of Mines in coöperation with the University of Utah is studying the possibilities of hydrometallurgical treatment of oxidized ores of zinc. Sulphide ores have been successfully treated by wet processes, but so far no commercial success has been attained in applying the hydrometallurgical process to the oxidized ores. In this study attention was given (1) to the technical problems involved in applying such treatment to oxidized ores, that are not so serious in treating sulphide ores, and (2) the economic desirability of wet treatment for ores not amenable to ordinary reduction methods, especially low-grade complex ores. Under (1) it was found that some of the processes tried seem to be commercially feasible for the ores tested. A proposed method that combines the most promising features was worked out. The results may be summarized as follows:

(1) Mixtures of oxide and sulphide ores with an oxidized content up to 70 per cent. yield sufficient water-soluble zinc on roasting to compensate for the acid radical used by the impurities in leaching.

(2) With a 5 per cent. acid leach liquor satisfactory zinc extractions are obtained without difficulties due to gelling of the silicate solutions.

(3) A satisfactory removal of iron and silica from the leach liquors can be made with 30 per cent. of the weight of the calcine leached, yielding solutions suitable for settling and filtration.

(4) A combined zinc extraction of 85.6 per cent. can be obtained by treating the precipitate from the neutral leach with 6 per cent. acid without excessive re-solution of the impurities.

(5) The purified solutions are suitable for electrolysis.

Under the second part of the study it was found that there are

large amounts of oxidized ores in the Rocky Mountain region which could be profitably treated by hydrometallurgical processes only in terms of high prices for zinc, or if a new process were installed.

A more detailed description of this work is presented in a recent publication issued as Bulletin 14 of the University of Utah.

An Experiment on the Origin of the Earth's Magnetic Field. H. A. WILSON. (*Proc. Roy. Soc.*, A 727.)—"Of the many suggestions that have been made as to the origin of the earth's magnetic field, perhaps the most promising is that it may be due to a slight modification of the laws of electrodynamics from the commonly accepted form. Electrically neutral matter is believed to consist of an intimate mixture of enormous amounts of positive and negative electricities, the electric and magnetic effects of which are usually supposed to balance each other. If the balance were not quite exact, then small residual effects would be expected, among which gravitation and the earth's magnetic field might be included. On such an hypothesis we might expect moving matter to produce a magnetic field similar to the field due to moving electricity." A calculation based on a reasonable hypothesis connected with the suggestion just quoted led to values of the magnetic field at the equator of the earth and likewise of the sun that are three times as great as the actual fields existing at the two places. To test by experiment the possibility of getting a magnetic field by the motion of matter, a bar of iron was swung as a pendulum, "the result of which seems to be to show that not more than one part in one thousand of it can be attributed to a magnetic effect of moving matter." The direct consequence of this experiment is to rule out translatory motion as having no marked effect. Other considerations show that motion of rotation cannot account for the earth's field. G. F. S.

Hydrogen Sulphide as a Poison.—The literature on the toxic action of hydrogen sulphide is reviewed by C. W. MITCHELL and S. J. DAVENPORT (*Public Health Rep.*, 1924, **39**, 1–13). Hydrogen sulphide is one of the most toxic gases, and is comparable to hydrogen cyanide (prussic acid) with respect to rapidity of action and concentration producing death. The action depends upon the concentration; a concentration of 0.005 per cent. is sufficient to produce poisoning. Continued exposure to a concentration of 0.02 per cent. during several days may produce death. The exact mechanism of the poisoning is still unknown. Since hydrogen sulphide may frequently be present in toxic concentrations in certain industries this gas must be recognized as an important industrial poison. J. S. H.

THE FRANKLIN INSTITUTE.

(Proceedings of Stated Meeting held Wednesday, April 16, 1924.)

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, April 16, 1924.

MR. WM. C. L. EGLIN, President, *in the Chair.*

The Board of Managers submitted their report. The report recorded the election of the following Resident Members:

Herman Carl Albrecht, B.S.,
Electrical Engineer,
Philadelphia Electric Company,
Philadelphia, Pennsylvania,

Henry P. Rust, B.A.Sc.,
Manager, Harper and Taylor, Engineers,
Philadelphia, Pennsylvania,

the following Non-resident Members:

Eskil Berg, E.E., M.E.,
Electrical Engineer,
General Electric Company,
Schenectady, New York,

Daniel E. Moran, Esq.,
Civil Engineer,
9 East Forty-fifth Street,
New York City, New York,

Charles H. Umstead, Esq.,
Civil Engineer,
Gulfport, Mississippi;

and additions to the Library by gift, 25 volumes, 134 pamphlets and by purchase 17 volumes.

The Chairman announced that at the last meeting of the Board of Managers, the Committee appointed to consider the exercises to celebrate the Centennial of the founding of the Institute, recommended a program which will consist of addresses and lectures by a number of distinguished foreign scientists and also by a number of men prominently identified with scientific technical and industrial work in this country. The Centennial will be observed September 17, 18 and 19 of this year.

The Chairman then announced that the next order of business would be the conferring of Honorary Membership on Dr. Charles A. Coffin, and asked Dr. Samuel Insull, a Life Member of the Institute to present Doctor Coffin.

Doctor Insull said:

"Mr. President, Ladies and Gentlemen, and Distinguished Guests, and Members of The Franklin Institute: It is a peculiarly personal privilege that I have this afternoon to perform in accepting the invitation of your President

to present the gentleman whom the Institute proposes to honor to-day. He was born of sturdy New England stock nearly eighty years ago in the State of Maine. His early business career was spent as a manufacturer in Lynn, Massachusetts. Becoming a Director of the Thomson-Houston Electric Company in 1882, he was elected its Vice-president in 1884, and as such was the active executive head of the Company until he effected its consolidation with the Edison General Electric Company in 1892.

"Since then he has been successively President and Chairman of the Board of the General Electric Company until his recent resignation from the latter position, and he is still very active in directing the destinies of that marvellous institution which is such a monument to his work and ability.

"The development of the electric light and power industry has been influenced to no small extent by the remarkable foresight and leadership of your guest of to-day. Without any special scientific or technical training, but with an intuition almost uncanny, his recognition of men of great scientific attainment and of purely scientific research and the part they play in industrial development is singularly illustrated by the Research Laboratory at Schenectady, the establishment of which was made possible by him.

"Not alone as a manufacturer and an administrator of a great industrial organization are we indebted to him. He was among the first to recognize the desirability of bringing the use of electric energy to every household and every workshop and a large portion of his career has been devoted to the development and financing of electric transportation companies, local electric light and power companies, and the establishment of vast power systems conveying electric energy from hydro-electric plants to distant points where such energy could be made subservient to the uses of mankind.

"He is a many-sided man. He has the highest possible conception of the duties of citizenship. He has the keenest appreciation of the arts, of literature and of nature. He has devoted much of his time and money to educational matters. His charities are so catholic in character that his right hand hardly knows what his left hand does.

"His work during the war in connection with the Red Cross and as head of the Franco-American War Relief Clearing House would of itself entitle him to distinguished recognition by his fellows. Modest beyond ordinary conception, desirous of giving credit at all times to others and claiming none whatever for himself, with a loveableness of character which endears him to his friends, and having 'a vision, courage and constructive ability signally contributing to the upbuilding of the great electrical and other industries'—it affords me the keenest personal pleasure to introduce the guest whom you propose to honor—Dr. Charles A. Coffin."

The Chairman then said:

"Doctor Coffin, upon the recommendation of the Board of Managers of The Franklin Institute, and upon the unanimous vote of the membership of The Franklin Institute, I have the honor, sir, to present to you Honorary Membership in the Institute, in recognition of your vision, courage and constructive ability, signally contributing to the upbuilding of electrical and other industries—basic factors in modern civilization. Doctor Coffin, I welcome you as an Honorary Member of The Franklin Institute."

In expressing his appreciation of the honor conferred upon him, Doctor Coffin said:

"Mr. President, Mr. Insull, Ladies and Gentlemen:

"I have no adequate phrase in which to express my appreciation and gratitude at being admitted to honorary membership in that venerable and distinguished organization, The Franklin Institute.

"To have received this unmerited honor and at the same time to have my very dear friend, Samuel Insull, a life member of the Institute, donor of The Franklin Medal, the leader and dean in his own great field of endeavor, come from afar to participate in the exercises, touch me very deeply.

"The generosity of the Institute and the courtesy of Mr. Insull will ever be a precious memory which shall never fade from my mind.

"In a lighter vein, may I add that having long had the highest regard for The Franklin Institute I am a little shaken in my confidence as to its entire wisdom, in that it has selected as the recipient of a great honor one who knows only enough about science to love the scientists, and only enough about the activities of The Franklin Institute to know that it exercises a wide and extraordinary influence and that it enjoys the homage and plaudits of the whole scientific world. I thank you."

The Chairman then announced that the paper of the day on "Compressibility, Internal Pressure and Atomic Magnitudes" would be presented by Dr. Theodore W. Richards, Director, Wolcott Gibbs Memorial Laboratory, Harvard University, Cambridge, Massachusetts.

Doctor Richards said:

"It is a very special pleasure to me to be here to-day. For a long time I wanted to come to this historic building, because of the far-spread reputation of The Franklin Institute; but especially during the last eight years I wanted to come here because of the very great honor which The Franklin Institute conferred upon me in 1916. I appreciated and appreciate this honor very highly indeed, and it is a great pleasure to be here to-day to say so. I appreciate it for many reasons: Because of the exalted reputation of the Institute; because of the names who are on that list of honor; because Philadelphia is my native place; and perhaps you can picture to yourselves what a pleasure it is to receive such a great mark of esteem from one's home people when circumstances have taken one away. I only wish that I deserved more adequately this honor.

"Now before beginning what I am going to say, I want to say one other word about the interesting development of The Franklin Institute. I had the pleasure this morning to see the wonderful laboratories which are being built on Nineteenth Street for purposes of physical and chemical and physico-chemical research. The metamorphosis which those old houses have received is really magical. They are laboratories planned with very unusual skill, insight and foresight for all sorts of work in physics and chemistry and in the borderland between. There is provision there for all the fundamental things which are necessary, both for chemists and for physicists, and there is room and flexibility for the addition of anything else that the particular investigation to be undertaken at any given time may need. The laboratories are quite unique, it seems to me, planned with admirable skill, and I congratulate The Franklin Institute on possessing them, and wish all success in the future to this really great undertaking.

"Now, Ladies and Gentlemen, we come to the subject which your Chairman has already announced—'Compressibility, Internal Pressure and Atomic Magnitudes.'"

Doctor Richards pointed out that the changes of volume suffered by liquids and solids during physico-chemical reaction are shown to bear systematic relation to the chemical affinity and cohesion brought into play. Greater affinity accompanies smaller volume, and greater contraction occurs, the greater the compressibility of the substances concerned. Hence it is inferred that chemical and cohesive affinity exert pressure, which tends to diminish the volume of the reacting systems. Apparent exceptions, on suitable interpretation, were shown to support the rule. From recent determinations of compressibility by the speaker and by Prof. P. W. Bridgman at Harvard, mathematical inferences were drawn as to the magnitude of the pressures thus produced. Moreover, since the changes of volume must be referred ultimately to the spaces occupied by atoms in solids and liquids, these considerations make it possible to compute (with a minimum of hypothetical speculation) the actual magnitudes of the bulks of individual atoms. The speaker endeavored to present the salient features of this train of thought in such a manner as to be comprehensible to those not conversant with the matter.

The Chairman said: "Gentlemen, I am sure that I am expressing your views when I say that the Institute deeply appreciates Doctor Richards' contribution to-day, and I would ask you to extend to him a rising vote of thanks."

Adjourned.

R. B. OWENS,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, April 2, 1924.)

HALL OF THE INSTITUTE,
PHILADELPHIA, April 2, 1924.

MR. GEORGE H. BENZON, JR., *in the Chair.*

The following reports were presented for final action:

No. 2779: Locomotive Feed Water Heater. The Edward Longstreth Medal to Thomas C. McBride, Esq., of Philadelphia, Pennsylvania.

No. 2824: The Louis Edward Levy Medal to Dr. Harvey Fletcher, of the American Telephone and Telegraph Company, New York City, for his paper on "Physical Measurements of Audition and Their Bearing on the Theory of Hearing," printed in the issue of the JOURNAL OF THE FRANKLIN INSTITUTE for September, 1923.

and the following reports were presented for first reading:

No. 2817: Duryea's Work in Early Automobile Development.

No. 2823: Sheen Tunnel Machine.

The Committee having in charge the investigation of the Locomotive Feed Water Heater reported in part as follows:

It is common practice in marine and stationary power plants to utilize to the

fullest extent possible, the otherwise waste heats in the stack gases and exhaust steam. That this might be done to a greater or lesser extent in locomotives has long been known and much work has been done to accomplish this end.

The ordinary locomotive type of injector is a simple and reliable device which performs the functions of a pump and of a heater and in so doing has a combined efficiency of substantially 100 per cent. Consequently, the conditions in locomotive practice demand that the apparatus for feed water heaters be kept within size limits, be easy to clean and repair. Changing conditions, especially in the matter of fuel costs, have increased the desirability of utilizing all possible waste heat from locomotives and the Worthington Locomotive Feed Water Heater is one that has been developed by Mr. McBride in response to this demand.

Briefly, the heater developed by Mr. McBride is mounted on the side of the locomotive and the unit consists of a steam pump directly connected to pumps respectively for cold water drawn from the tender, and for hot water to be delivered to the boiler. The cold water drawn into the unit is mixed with exhaust steam in a chamber in the heater, from where it is pumped to the boiler as required.

The use of this heater on locomotives results in a saving of fuel and economy in the use of water due to the utilization of the condensate and heat in the exhaust steam. It also reduces boiler maintenance as it acts as a deaërator, reducing oxidization, and in addition a part of the incrusting solid in the feed water are deposited in the heater where they may be readily removed.

The Sub-committee on Literature reported in connection with the award to Doctor Fletcher that:

> "By the utilization of apparatus which permits the translation, without distortion, of sound vibrations into electrical vibration, Doctor Fletcher has studied quantitatively the nature of speech and hearing. He has made a precise analysis of speech sounds into their frequency and volume characteristics. He has accurately determined quantitatively the performance of both normal and abnormal ears. This work is of fundamental importance in the design of telephone circuits and apparatus. It also makes a strong humanitarian appeal because of its bearing upon the diagnosis of and methods of alleviating defective hearing.
>
> "Doctor Fletcher's work constitutes a signal advance in this important field of science."

R. B. OWENS,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, April 9, 1924.)

RESIDENT MEMBERS.

HERMAN CARL ALBRECHT, B.S., Electrical Engineer, Philadelphia Electric Company, Philadelphia, Pennsylvania.

HENRY P. RUST, B.A.Sc., Manager, Harper and Taylor, Engineers, Philadelphia, Pennsylvania.

NON-RESIDENT MEMBERS.

ESKIL BERG, E.E., M.E., Electrical Engineer, General Electric Company, Schenectady, New York.

DANIEL E. MORAN, ESQUIRE, Civil Engineer, 9 East Forty-fifth Street, New York City, New York.

CHARLES H. UMSTEAD, ESQUIRE, Civil Engineer, Gulfport, Mississippi.

CHANGES OF ADDRESS.

MR. FRANK M. ASHMEAD, 3016 Euclid Avenue, Cleveland, Ohio.

MR. ROBERT ATKINSON, 312 Gowen Avenue, Mount Airy, Philadelphia, Pennsylvania.

MR. J. SNOWDEN BELL, Box 0316, City Hall Station, New York City, New York.

MR. RICHARD C. CHARLTON, 5017 Griscom Street, Frankford, Philadelphia, Pennsylvania.

MR. THEOBALD F. CLARK, 490 Abington Avenue, Chestnut Hill, Philadelphia, Pennsylvania.

MR. HENRY FRED COLVIN, 2ND, 754 Lexington Avenue, Brooklyn, New York.

MR. GEORGE L. COPPAGE, Production Manager, The Pusey and Jones Company, Wilmington, Delaware.

MR. W. L. DEMPSEY, Thum Engineering Company, 406 Market Street, St. Louis, Missouri.

MR. J. M. DIVEN, Superintendent of Water Works, Troy, New York.

MR. HALCOLM ELLIS, 16 Lawrence Street, Newark, New Jersey.

MR. W. L. GARRELS, Box 270, Rural Route 13, Kirkwood, Missouri.

MR. HENRY CLAY GIBSON, Jenkintown, Pennsylvania.

MR. JOHN J. GIBSON, Westinghouse Commercial Investment Company, 150 Broadway, New York City, New York.

MR. JOHN D. GILL, 260 South Broad Street, Philadelphia, Pennsylvania.

MR. J. H. GRANBERY, 234 West Twenty-first Street, New York City, New York.

MR. ALFRED B. HITCHINS, 33 West Sixtieth Street, New York City, New York.

MR. RICHARD HOWSON, 32 South Broad Street, Philadelphia, Pennsylvania.

MR. E. E. KELLER, Braywood, Quarton Road, Birmingham, Michigan.

MR. O. W. KETCHAM, 125 North Eighteenth Street, Philadelphia, Pennsylvania.

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BOOK REVIEWS.

COLLOID CHEMISTRY. Wisconsin Lectures. By The Svedberg. 265 pages, 115 illustrations, 8vo. The Chemical Catalog Company, New York, 1924. Price, $3.

This is one of the series of monographs in course of publication by the American Chemical Society and is uniform in style and form with those already issued. It is an issue of the lectures delivered by Svedberg in the spring and summer of 1923 at the University of Wisconsin. On that occasion a symposium on the applications of colloid chemistry was held, most of the contributions to which have appeared in a separate volume already reviewed in this JOURNAL. The field to which the volume in hand is devoted has become of vast interest and importance to chemistry and physics, both pure and applied. An immense literature has developed within a comparatively few years. We are, of course, not astonished to learn that the properties and nature of finely divided materials suspended in fluids attracted attention in the early days of scientific chemistry. In an interesting review of the history of the subject the author quotes from

Macquer in 1774, who states that the "gold tinctures" (then much in vogue as medicinal agents) are nothing but finely divided metal. Berzelius pointed out that the yellow liquid obtained by treating arsenous oxide with hydrogen sulphide without addition of acid is really a "suspension of transparent particles." Graham is, of course, noted as the leader in the early development of this field. He introduced the words "crystalloid" and "colloid" and made many experiments along the line. Among the results of his work was the discovery of the principle of dialysis, and it is interesting to note that the original dialyser is still preserved in London.

In the book special attention has been given to the technique of colloid chemistry, much space being devoted to electrokinetic phenomena. Emphasis is laid on quantitative methods, it being the author's opinion that real advance is possible only by developing methods of accurate measurements. Indeed, he asserts that by pursuing purely qualitative methods more confusion than progress has resulted. The thought occurs that in this respect colloid chemistry may be following the evolution of chemistry at large, for Kopp's fourth period of the history of chemistry is the era of quantitative analysis. Necessarily, prominence has been given to the results of work done in the author's laboratory, an entirely appropriate use of space.

A vast amount of literature in books and journals has been produced in the last decade or so on the subject of colloids, yet it must be acknowledged that the older chemists are not at all clear as to the nature and significance of this line of research. There seems still to be lacking a work which shall take up the question in an elementary way, not exactly a book of words of one syllable, but at any rate much less complicated than most of the available material. Perhaps the difficulties that chemists experience with this subject, as with the newer views on the nature of the atom, is because the majority of investigators are physicists rather than chemists. The vast mass of chemists are engaged either in routine analyses or in teaching the more elementary phases of the science. To them the structure of the atom and the nature of the colloid particle are not directly interesting or even important and they ignore the whole field on account of the complexity which it presents. It has sometimes occurred to the reviewer that an illuminating illustration might be made by comparing colloids to a street mob and crystalloids to a military company. As the former rushes down the street with no definite order nor organization it gathers additions from the bystanders (by adsorption), but the military company, organized and definitely arranged, loses none of its components and takes no one in from those who stand along its route.

A few minor typographic errors are noted. "Spark" is occasionally spelled "sparc," and on page 100 "hundreds" should be "hundredths." Several pages are devoted to the structure of the silver halide emulsions which are the basis of common photographic processes. The problems have been studied extensively in the laboratory of the Eastman Company, but no attention appears to have been given to the fact that after exposure of the plate, the silver salt may be completely removed by thiosulphate and yet a perfect image be obtained by a simple process of development. Here is a field probably for ultramicroscopic study.

HENRY LEFFMANN.

FOIBLES AND FALLACIES OF SCIENCE. By D. W. Hering, New York University. vii–294 pages, many illustrations, small 8vo. New York, D. Van Nostrand Company, 1924. Price, $2.50 net.

This is a very interesting book. It gives well-written accounts of many vagaries and frauds that have been practised upon human beings during the ages. Some of these have been wide-spread and have rested for their popularity upon the cupidity of people. Such, for instance, is alchemy. The desire to convert the baser metals into the precious was not only attractive to people at large, but to rulers whose treasuries were exhausted, or who desired opportunities for larger expenditure than their dominions afforded. Kopp, the famous writer on chemical history has written extensively on this phase of metallurgy. Other portions of Professor Hering's book are devoted to accounts of impositions exploited by particular individuals, such as the Keely motor and Redheffer's perpetual motion machine. The museum of The Franklin Institute has a model of the Redheffer machine made under the direction of Nathan Sellers, whose son, Coleman, had discovered some mechanical conditions in the machine as exhibited which indicated that it was moved from an outside force. The model, which had a clockwork motor, worked so well that Redheffer was himself deceived and offered Mr. Sellers a share in the exploitation of the machine.

The Keely motor was one of the most remarkable frauds ever perpetrated. Unlike some of those described in this book which were exploited at a time when physics and chemistry were but imperfectly developed, and scientists had no satisfactory means of reaching the public and warning against silly claims and statements, Keely began his long career in a community in which both pure and applied science had been extensively cultivated for more than a century. His shop was located in Philadelphia, the city in which Franklin, Hare, Fitch and Evans had worked, and in which a host of able, worthy successors, were then engaged. It would be a great wrong, however, to assert that these contemporary scientists had failed to see fraud in Keely's claims. They proclaimed their lack of faith in the motor, and Keely took care that no person who had any real mechanical or chemical knowledge should see more than the outside of the machinery. He allowed scientists whose accomplishments were in an entirely different field to visit his shop, and sometimes, as might be expected, imposed on them. Dr. Joseph Leidy was thus inveigled and unfortunately made a statement which was twisted into an endorsement of Keely's claims. Doctor Leidy was a naturalist and especially a zoologist of the highest accomplishment. He could not have been fooled by any geologic or zoologic fake (the Cardiff Giant, for instance), but he was easily imposed on by a complicated piece of machinery and voluble expression of physical and chemical terms. Portions of the tubes which carried the compressed air that was one of the methods used for moving the Keely machinery are now in The Franklin Institute museum. This comparison leads the reviewer to remark that the title of this book is unfair. The stories given are not accounts of the fallacies and foibles of science, but of ignorance and superstition, and the evidences of human dishonesty.

The book is liberally illustrated and affords the ordinary reader who is interested in the progress of science an opportunity to learn something of the

misuses to which human ingenuity has been put. Among the specially interesting chapters is one on the attacks that have been made on standard theories in physics, such as the vibratory theory of light, the wave theory of sound and an extended account of the so-called "water-witching," detecting the presence of water (and valuable ores) by means of the divining-rod. The trend of the author's opinion on the last topic is adverse to the claims of the diviners, but it is a knotty problem. Like some so-called spiritualistic phenomena, there is a great deal of fraud and mere coincidence, but the data at hand seem to show a small residuum of results not easy of explanation on known scientific principles.

Professor Hering's book deserves wide circulation, and it is to be hoped that future editions will bear a title juster to science than that which it now bears.

HENRY LEFFMANN.

ABRIDGED SCIENTIFIC PUBLICATIONS FROM THE RESEARCH LABORATORY OF THE EASTMAN KODAK COMPANY. Vol. 6, 1922, paper. vii–238 pages, 8vo. Rochester, New York, Eastman Kodak Company, 1923.

A gratifying phase of industrial development in the United States is the attention given by large corporations to research in pure science, as well as in science directly applicable to the work in which they are respectively engaged. It must, of course, be expected that even the studies in pure science will be associated in the lines in which the company is engaged. The Eastman Kodak Company has done a great deal of good work, outside of its immediate problems and interests, in establishing a department for the manufacture of synthetic chemicals, putting in the hands of American chemists the prompt obtaining of many materials for research and analytic use. The volume in hand contains thirty papers relating to problems in photography. As noted in the title, the papers are abridged, but with each the reference is given to the journal in which the article appeared in full. In these references, the commendable method is followed of giving the year as well as the volume. The reviewer, however, prefers strongly the method of giving the year at the beginning of the reference and printing all figures except the volume number in the regular font of the text, distinguishing the volume in Arabic numerals with a distinctly bold face. Some international agreement should be reached as to the method of stating references and also for abbreviations of the titles of journals. Space may be saved by using initials when these are not ambiguous. JACS is quite sufficient for the *Journal of the American Chemical Society* and JSCI indicates only the *Journal of the Society of Chemical Industry*. The Eastman Kodak Laboratory wisely holds that it is better to publish full papers in journals devoted to the field to which the paper relates, than to publish a bulky volume privately issued, but it is also good policy to collect in abstract form, as the company has done, the contributions and place them in one volume with references to the place of publication of the full papers. Those interested in the problems of photography will find valuable and interesting matter in this book, and the company that has made "kodak" an international word deserves thanks for publishing it.

HENRY LEFFMANN.

THE ART OF LETTERING. By Carl Lars Svenson. With 96 full-page plates and 125 pages of text. 4to. New York, D. Van Nostrand Company, 1924. Price, $3.50.

The origin of alphabets is still undetermined. Remarkable evidence of artistic skill on the part of primitive man has been brought to light in recent years by the investigations of the caves in western Europe, but no inscriptions are seen indicating a written language. When and where letters were first devised are problems of intense interest, but it is doubtful if the data will ever be obtained. We have methods of writing of high antiquity. The Egyptian hieroglyphics, the cuneiform characters, the Semitic and Chinese systems are well known and ancient. Of all letter forms, the Roman seems to lend itself best to artistic modifications, and in the countries in which this is used as the regular type much ingenuity and skill have been shown in securing striking and ornamental forms. The clearness and convenience of the Roman letter have appealed even to the nations which use German text in popular works, for all scientific publications in Germany and Austria appear in the Roman letter. The German letters, together with their cognate forms "Old English," are not without some artistic value, but this is limited. Products of the perverse ingenuity of the scriveners of the middle ages, they may well be limited to-day to a few special uses.

The author of the book calls attention to the fact that while lettering is primarily intended to convey information or to record data, it has developed an ornamental phase which is of great vogue and usefulness. The plates give illustrations of many varieties of lettering and the text gives instruction in the methods of writing with different forms of pens. The origin of our present alphabet is ascribed by the author to a simplication of the picture language of the ancient Egyptians, presenting essentially the familiar view that the Phœnicians were the intermediary of transmission to the Greeks and Romans.

> "You have the letters Cadmus gave,
> Think ye he meant them for a slave?"

One wonders, in passing, what Byron would say if he could see the nation for which he made the supreme sacrifice now becoming a republic, but this thought is apart from the question of the work in hand. Description can do little towards indicating the merits of the volume. It must be examined to be appreciated and notwithstanding the necessarily disconnected and fragmentary character of the plates, they make interesting reading. It is a fine specimen of selection and presentation of the many phases of artistic opportunity in lettering and incidental illustration and the publisher and printer have done their duty by the text and plates. HENRY LEFFMANN.

THE PATENT OFFICE: ITS HISTORY, ACTIVITIES, AND ORGANIZATION. By Gustavus A. Weber. xii–127 pages, 8vo. Baltimore, The Johns Hopkins Press, 1924. Price, $1.

This is a volume of the Service Monographs of the United States Government, prepared under the auspices of the Institute for Government Research, being No. 31 of the series. It relates to a department which has been especially

active in the history of the nation. It was established under a brief and simple clause of the Constitution, giving power to Congress "To promote the progress of science and useful arts by securing for limited times to authors and inventors the exclusive rights to their writings and discoveries." The people of this country developed great activity and versatility in mechanical inventions and this phase has been a characteristic among nations. We have not, however, occupied so prominent a place in the domain of pure science or in the applications of chemistry. Germany's prominence in the latter field has been manifested for more than half a century, and is still considerable, although the nations which were allied against her in the late war have been induced by sad experience to exert themselves to better development, especially in certain key industries.

Congress took early advantage of its powers in regard to patents, and in 1790 at the suggestion of Washington the first law was passed. The nation had in its formative period two men who were especially strong in their desire to promote science, Jefferson and Franklin. Thomas Paine was also deeply interested in the same line, but at the time that the Constitution was framed and when it was first put into effect, he was interested more strongly in political matters, especially the struggle for general freedom. Thirty-one pages of the book are devoted to a history of the development of the Patent Office, after which follow sections on Activities and Organization. At present the department is extremely active, the number of patents taken out in the course of a year being very large.

The series to which this volume belongs is intended to inform the public of the character and methods of the several departments. The United States Government is probably the largest business enterprise in the world. The budgets of some of the other nations may be as large or larger, but the money is much more devoted to salaries and grants to rulers and their families. It is, therefore, useful and interesting to have accurate data as to the operations of the major departments of our nations. HENRY LEFFMANN.

SOLUBILITY. By Joel H. Hildebrand, Ph.D., University of California. 206 pages, numerous diagrams, 8vo. New York, The Chemical Catalog Company, 1924. Price, $3.

This is one of the Monograph Series of the American Chemical Society, in which many volumes of useful information have been published. Doctor Hildebrand has devoted a great deal of attention to the problems of solubility and presents in this book the results of experiment and reading. Dictionaries of solubilities have been long known in chemical literature, but they have been merely statistical, that is, presenting data of degrees of solubility without any discussion of theories. They have been necessarily almost entirely compilations. In many cases, the reports of investigators have been discordant and the compiler has been unable to determine the actual figures. The present work is not of this type, but an elaborate study of the physical chemistry of solubility. Much space is given to a discussion of the relations of Raoult's law. In defining solution, attention is called to the fact that in many cases at least, the terms "solute" and "solvent" are interchangeable. Common ammonia (ammonium hydroxide solution) is regarded as a solution of gaseous NH_3 in water, but it

may with justice be regarded as a solution of water in liquid ammonia. This interchangeability is still more evident in the case of common ether and water, with which we may have the condition of an ethereal solution of water or a water-solution of ether. In most cases the solvent is the substance present in greatest amount, but a solution of naphthelene in benzene at 70° contains far more of the solid component than of the liquid. Mortimer has defined the solute as the compound which separates first on cooling the system. Hildebrand insists that our ordinary views should not allow us to overlook the fact that the functions of solvent and solute are interchangeable. From these simple considerations the author passes to the presentation of the complex physical chemistry and then takes up the ideal solution and Raoult's law. This is considered from many points of view, many pages being devoted to the subject. Solubilities of gases, of solid non-electrolytes and solid electrolytes receive extensive treatment. By the collection of an immense number of data and comparison of them, the laws of solubility have been determined, and the work becomes a valuable compilation to a special and important field in physical chemistry. It is, of course, highly technical and under the present tendencies of chemical science will be appreciated by only a comparatively limited number of readers, but it is a worthy contribution to American literature and is acceptable as an evidence that not all our energies are devoted to directly practical objects.

HENRY LEFFMANN.

ANILINE AND ITS DERIVATIVES. By P. H. Groggins, B.S. 256 pages, 29 illustrations, 8vo. D. Van Nostrand Company, New York, 1924. Price, $4.

A book of this type is doubly interesting. It is an agreeable evidence of the development that our country is making in securing independence in the coal-tar industries, and it is a striking example of high specialization that this industry has reached. There is a sad story connected with the earliest developments of the utilization of coal-tar, for Mansfield, Hofmann's assistant, lost his life in the course of some experiments in fractional distillation, but perhaps a sadder story is to consider the awful results that followed British indifference to the value of the industry. The initial point in the tar industries may be considered as Faraday's discovery of benzene from the compression of coal-gas. To this followed Hofmann's and Mansfield's isolations of the hydrocarbon and its higher homologues from the commercial tar. Aniline itself came into notice from an entirely different source, from which it also obtained its familiar name. It was as a component of the liquid resulting from the destructive distillation of indigo that Fritsche detected and named it from the specific name "anil" of the plant producing the dyestuff. The word originally the Sanskrit "nila," dark blue, passed into Arabic as "Al-nila" and was shortened into "anil." Hofmann (whose name, by the way, our author always spells "Hoffman") obtained aniline by distilling indigo. It is now generally known that Perkin was endeavoring to make quinine and in the course of his investigations oxidized aniline sulphate with potassium dichromate. In the black mass that resulted he found a purplish substance which he afterwards manufactured on the large scale as a dyestuff, the first of the "aniline" colors. Coal-tar colors are now known in great number and many of them are not made from aniline, but this title is a popular one. The opportunities of a great industrial development were

open to British capitalists, but they failed to take advantage of these, and the sceptre of the coal-tar industry passed to Germany and the late war showed what a tower of strength the industry is. Notwithstanding the efforts made and making to compete with the German industries, the outlook, while encouraging, is by no means unreservedly gratifying. Very lately the French authorities have made special arrangements to take over patents and secrets of the synthetic ammonia process, and have officially stated that British, French and American experts have failed to solve the catalytic problems of these processes. Not only are the coal-tar industries of Germany important both in war and peace, but they are, under normal conditions, exceedingly profitable, for the average annual dividends paid by leading firms is about 20 per cent. W. M. Gardner, in his work "The British Coal-tar Industry," gives many data concerning the development and decline of the British phase.

The work in hand deals with the manufacture of aniline from its now invariable source, nitrobenzene, and the manufacture of many derivatives. Aniline has a large family of derivatives most of which are directly or indirectly employed as dyestuff intermediates, but a few, such as acetanilid, are used in medicine. Though intended as an elaborate and complete treatise for the chemical engineer, an object which it well fulfils, theory is not ignored, and much space is given to elucidating the reactions involved in the transformations. One notes with pleasure that the author knows how to express a benzene-ring symbol, a knowledge which some American chemists do not possess. A striking result of practical operations is the discovery that only about one thirty-sixth of the amount of hydrochloric acid that theory would require is actually used in the reduction of nitrobenzene by iron.

Hygiene is a very important phase of modern industrial operations and the aniline industry is one of the most serious sources of poisoning. A chapter is devoted to this subject; in which many valuable facts are given and the diagnosis and treatment of cases stated. In the course of this chapter an account is given of the apparent incidence of malignant (that is, cancer) tumors of the bladder in workers in organic chemical industries. The data are from a German source.

Mild criticism must be made of some errors. The error in Hofmann's name has already been noted. Bibliographic references are in several places carelessly expressed. On page 7 there is a reference to "Zeitschrift." The periodical intended is the *Zeitschrift für angewandte Chemie*, but as there are over a score of publications in Germany with the title "Zeitschrift," a more precise designation is needed. Similarly, on page 137 there is a reference to "J. I. C." This is intended for J. I. E. C., the well-known journal issued by the American Chemical Society. On the other hand, the title "*Berichte der deutschen chemischen Gesellschaft*" is in one place given in full, although "Ber." is now generally considered fully sufficient to indicate this important publication. Perhaps the best method of quoting references is by a consecutive series of numbers with the titles, giving name, suitably abbreviated, series, if any, year, volume and page, in an appendix. The book is neatly printed, well illustrated, and is a timely contribution to the development of American coal-tar industry now so important to us as strength in peace and war. HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report No. 182, Aerodynamic Characteristics of Airfoils—III, Continuation of Reports Nos. 93 and 124. 43 pages, illustrations, quarto. Washington, Government Printing Office, 1924.

This collection of data on airfoils has been made from the published reports of a number of the leading aerodynamic laboratories of this country and Europe. The information which was originally expressed according to the different customs of the several laboratories is here presented in a uniform series of charts and tables suitable for the use of designing engineers and for purposes of general reference.

It is a well-known fact that the results obtained in different laboratories, because of their individual methods of testing, are not strictly comparable even if proper scale corrections for size of model and speed of test are supplied. It is, therefore, unwise to compare too closely the coefficients of two wing sections tested in different laboratories. Tests of different wing sections from the same source, however, may be relied on to give true relative values.

The absolute system of coefficients has been used, since it is thought by the National Advisory Committee for Aeronautics that this system is the one most suited for international use and yet is one for which a desired transformation can be easily made. For this purpose a set of transformation constants is included in this report.

Each airfoil section is given a reference number, and the test data are presented in the form of curves from which the coefficients can be read with sufficient accuracy for designing purposes. The dimensions of the profile of each section are given at various stations along the chord in per cent. of the chord, the latter also serving as the datum line. When two sets of ordinates are necessary, on account of taper in chord or ordinate, those for the maximum section (at centre of span) are given on the individual characteristic sheets, while those for the tip (dotted) section are given in separate tables. Where the ratio of ordinate to chord remains constant the one set of ordinates applies to both centre and tip section. The shape of the section is also shown with reasonable accuracy to enable one to more clearly visualize the section under consideration, together with its characteristics.

The authority for the results here presented is given as the name of the laboratory at which the experiments were conducted, with the size of model, wind velocity, and year of test.

POPULAR SCIENCE TALKS. A series of popular lectures on interesting topics. 216 pages, illustrated, paper, small 8vo. Philadelphia College of Pharmacy and Science. Price, $1.

Popular lectures on a great variety of scientific subjects have been a prominent feature of educational activity in large cities for a considerable time. The Philadelphia College of Pharmacy and Science entered the field in the session of 1922–23, and in the succeeding session made a step in a new direction by publishing the lectures in book form. A neat, paper-bound volume has been issued, which contains twelve lectures, covering a wide variety of subjects. The college has a large and active instructional corps, and the lectures are the results of special investigations in the many fields that the teaching

work of the institution includes. Several of the lectures are fully illustrated, among which may be mentioned Doctor Youngken's on animal-eating plants, LaWall's on the romance of spices, and Cook's on the making of medicines. The volume forms an interesting summary of applied science in many important fields.

PUBLICATIONS RECEIVED.

Aniline and Its Derivatives, by P. H. Groggins, B.S. 256 pages, illustrations, 8vo. New York, D. Van Nostrand Company, 1924. Price, $4.

Solubility, by Joel H. Hildebrand, Ph.D. American Chemical Society Monograph Series, 206 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1924. Price, $3.

Line Charts for Engineers, by W. N. Rose, B.Sc. 95 pages, illustrations, 8vo. New York, E. P. Dutton and Company, 1923. Price, $3.

Traité Élémentaire des Nombres de Bernoulli, par Niels Nielson. 398 pages, 8vo. Paris, Gauthier-Villars et Cie., 1923. Price, 50 Francs.

Barrier Charts for Health Officers. A synopsis of preventive measures against communicable disease, in four tables, by S. H. Daukes. Four charts and introduction, folded to narrow octavo. London, Wellcome Bureau of Scientific Research, no date.

Canada, Department of Mines, Mines Branch. Summary Report on Mines Branch Investigations during the Calendar Year Ending December 31, 1922. 273 pages, illustrations, 8vo. Ottawa, King's Printer, 1924.

Steinmetz and His Discoverer, by John T. Broderick. 23 pages, portrait, 12mo. Schenectady, New York, Robson and Adee, 1924. Price, 50 cents.

U. S. Bureau of Standards: Circular No. 145, Summary of Technical Methods for the Utilization of Molasses. Collated from patent literature. 72 pages, illustrations, 8vo. Price, 15 cents. Circular No. 163, United States Government Specification for Titanium Pigment, Dry and Paste. 11 pages, 8vo. Washington, Government Printing Office, 1924.

National Advisory Committee for Aeronautics: Technical Notes, No. 184, Note on Vortices and on Their Relation to the Lift of Airfoils, by Max M. Munk. 14 pages, illustrations, quarto. No. 185, The Influence of Inlet Air Temperature and Jacket Water Temperature on Initiating Combustion in a High-speed Compression Ignition Engine, by Robertson Matthews and Arthur W. Gardiner. 5 pages, illustrations, quarto. Testing Airplane Fabrics, by A. Pröll. 29 pages, illustrations, quarto. Washington, Committee, 1924.

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